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An analytic study of TTF of standing wave RF gap based on Bessel–Fourier expansion ☆

Pei-Yong Jiang [a,b], You-Jin Yuan [a,*], Xue-Jun Yin [a], Peng Li [a], Chao Li [a,b], Zhi-Jun Wang [a]

[a] Institute of Modern Physics (IMP), Chinese Academy of Sciences (CAS), No. 509, Nanchang Road, 730000, Lanzhou, Gansu, PR China
[b] University of Chinese Academy of Sciences, Beijing 100049, China

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ABSTRACT

Transit time factor (TTF) is important in design and simulation of standing wave RF gaps. The TTF is usually constructed on the basis of a square wave model, and it is always expanded as a function of reduced velocity and structure factors. In order to express the particle's motion more authentically, the TTF is studied based on the Bessel–Fourier (B–F) expansion which is realized in BEAMPATH code. By expanding square wave electric field into harmonic electric fields, the voltage component and the TTF component of each order are obtained from corresponding harmonic electric field. The ratios of each order of voltage components and integral voltage form the weights working as coefficients of TTF components. Consequently, the effective resultant TTF depends on not only the particle's velocity and the structure factors, but also the RF phase the particle experiences. Simple expressions are derived after simplifying the complicated TTF equation in this paper.

1. Introduction

In general, a particle experiences a time-varying electric field in a standing wave RF gap. With a similar equation in a traveling wave guide for acquiring energy gain, transit time factor (TTF) is introduced to calculate effective voltage, while the synchronous phase is defined as the RF phase when the synchrotron particle passes through the center of a gap. The energy gain Δw is as follows:

$$\Delta w = qV_0 T \cos(\phi) \tag{1}$$

where q is particle charge per nucleon, V_0 is the integral voltage along the gap, ϕ is synchronous phase and T is the TTF.

As is given in Refs. [1–3], TTF is a derived simple equation. Meanwhile, since particles with different velocity experience different electric fields, the TTF is a function of reduced velocity (ratio of velocity to the speed of light) β, the high order terms of TTF are studied [4] and applied in the design of RF gaps [5,6].

Electric field can be obtained by solving the Poisson equation with PIC method for a complicated structure such as a tank, which needs to wait for a long time. In order to expedite calculation speed, quasistatic approximation is used when the geometrical variations are small compared to the free-space wavelength [2]. Under this condition, the Poisson equation changes into a Laplace equation with boundary conditions and analytical solutions of the electric field in a RF gap exist. The analytical expansions such as Bessel–Fourier (B–F) expansion are usually used to obtain electric field and then to simulate beam dynamics by tracking in codes. This paper focuses on TTF with B–F expansion method used in BEAMPATH code [7].

Setting the structure center as the original, a derived TTF equation is obtained from Eq. (1) as follows:

$$T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(\omega t(z)+\phi)\,dz}{\cos(\phi)\int_{-L/2}^{L/2} E(0,z)\,dz} \tag{2}$$

$$T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(\omega t(z))\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz} - \tan\phi \frac{\int_{-L/2}^{L/2} E(0,z)\sin(\omega t(z))\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz} \tag{3}$$

where L is the cell length, ω is the circular frequency of RF oscillation, and ϕ is the synchronous phase.

In most practical cases, the change of particle velocity in the gap is so small compared with the initial velocity that it can be ignored, leading to the outcome that the sizes of the drift tubes on both sides of a gap are similar when the accelerator works on a π mode. It results in the issue that the electric field distribution in the gap is close to being an even function with respect to a proper chosen original point which is usually the electric field center and also the structural cell center [1,2]. Consequently, the first term of Eq. (3) is dominant and the second term of Eq. (3) is close to zero. That Eq. (3) degenerates to its first term is regarded as the first approximation of TTF. Based on the first approximation, Ref. [1] describes the series form of TTF.

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* Corresponding author.
E-mail address: yuanyj@impcas.ac.cn (Y.-J. Yuan).

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However, under the condition of those structures which do not work on a π mode exactly such as APF [9] and KONUS [10], the lengths of the two neighboring drift tube lengths are usually different because of the difference of the neighboring synchronous phases. The asymmetric structure generates asymmetric electric field, and there is no proper electric field center as the original point any more, subjected to the assumption that the electric field is an even function and then the second term of Eq. (3) can be neglected. Thus the first approximation could not express the energy gain properly unless the second term of Eq. (3) is taken into consideration. For ease of calculation, Eq. (2) is used in this paper while the original point is chosen at the starting point of one cell because of asymmetry, and the integration path in Eq. (2) changes from $[-L/2, L/2]$ to $[0, L]$.

2. TTF models based on symmetric electric field

The second term of Eq. (3) is zero for a symmetric electric field. Assuming $E(0,z)$ is a square wave, the TTF is obtained from the first term of Eq. (3) [1–3]:

$$T = \frac{\sin(\pi g/\beta\lambda)}{\pi g/\beta\lambda} \tag{4}$$

where λ is wave length in free space, and g is the gap length. In practice, a square wave is assumed only between the tube walls where the radius equals the tube aperture a. Consequently, the TTF changes to be [2]:

$$T = \frac{\sin(\pi g/\beta\lambda)}{\pi g/\beta\lambda}\frac{1}{I_0(\mu a)} \tag{5}$$

where

$$\mu = 2\pi/\gamma\beta\lambda \tag{6}$$

where γ is the Lorentz factor. Eq. (5) is used in a code for beam dynamics calculation of a booster in RIKEN [8].

3. TTF of electric field with B–F series expansion

Using B–F expansion, the electric field along the axial direction E_z of a cell can be written in a series expansion [7] as

$$E_z = \sum_{m=1}^{M} E_{zm} \tag{7}$$

where E_{zm} is m-th order electric field in the cell:

$$E_{zm} = \frac{4U}{I_0(\mu_m a)\Gamma}\frac{\pi m(l+g)}{\Gamma}\frac{\sin\left[\frac{\pi m(l+g)}{\Gamma}\right]}{\frac{\pi m(l+g)}{\Gamma}}\frac{\sin\left[\frac{\pi m g}{\Gamma}\right]}{\frac{\pi m g}{\Gamma}}\sin\left(\frac{2\pi m z}{\Gamma}\right) \tag{8}$$

where l, d and g are the lengths of the former tube, the post tube and the gap, respectively, which are illustrated in Fig. 1. U is the integral voltage along the whole cell, and

$$\Gamma = l + d + 2g = 2L \tag{9}$$

$$\mu_m = \frac{2\pi}{\lambda}\sqrt{\left(\frac{m\lambda}{\Gamma}\right)^2 - 1}. \tag{10}$$

In the original derivations of Ref. [7] E_{zm} is proportional to certain field amplitude E_0 which is substituted by U/g in this paper.

Fig. 2 shows field components for the first and second terms and the resultant electric field distribution along a cell. In this example, the resonant frequency f is 200 MHz; l, d, g and a equal to 0.0638 m, 0.0611 m, 0.0312 m and 0.006 m, respectively; U is 330 KV; and the maximum order M is 10.

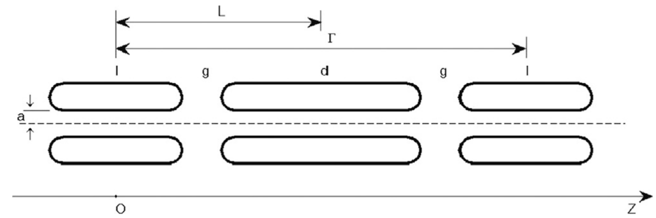

Fig. 1. Schematic diagram of one cell. l, d and g are the lengths of the former tube, the post tube and the gap, respectively. a is the tube aperture. The cell begins with the center of the former tube where it is set as the original point.

According to Eq. (8), the even order electric field is an odd function and the odd electric field is an even function. Integrate E_{zm} along a cell to obtain m order voltage component U_m. Consequently, U is a combination of M term voltage components:

$$U = \sum_{m=1}^{M} U_m$$

where

$$U_m = \begin{cases} 0, & m \text{ is even} \\ \dfrac{4U\sin\left(\frac{\pi m(l+g)}{\Gamma}\right)\sin\left(\frac{\pi m g}{\Gamma}\right)}{\pi^2 m^2 I_0(\mu_m a)}\dfrac{\Gamma}{g}, & m \text{ is odd}. \end{cases} \tag{11}$$

Under the assumption that the particle velocity change is ignored, the even term electric fields have no impact on the energy gain. Therefore, the order m is odd in this paper unless there is a special statement.

Assuming the RF phase that one particle passes through the center of a gap is ϕ, the RF phase when the particle reaches position z is

$$\phi_z = \phi - \frac{(l+g)\pi}{\beta\lambda} + \frac{2\pi z}{\beta\lambda} \tag{12}$$

and the phases that the particle entering and exiting the cell, ϕ_i and ϕ_f, are $\phi-(l+g)\pi/\beta\lambda$ and $\phi+(d+g)\pi/\beta\lambda$, respectively.

Using Eqs. (7)–(12), the TTF can be written in a series expansion as follows:

$$\begin{aligned} T &= \frac{1}{\cos(\phi)U}\int_0^L \sum_{m=1}^{M} E_{zm}(0,z)\cos(\phi_z)\,dz \\ &= \frac{1}{\cos(\phi)U}\sum_{m=1}^{M}\int_0^L E_{zm}(0,z)\cos(\phi_z)\,dz \\ &= \sum_{m=1}^{M}\frac{U_m}{U}\frac{\int_0^L E_{zm}(0,z)\cos(\phi_z)\,dz}{\cos(\phi)U_m} \\ &= \sum_{m=1}^{M}\frac{U_m}{U}T_m \\ &= \sum_{m=1}^{M} w_m T_m \end{aligned} \tag{13}$$

where $w_m = U_m/U$ is a weight factor, and T_m is the TTF of m-th order electric field. Substituting Eq. (8) into Eq. (2), the specific analytic expression of T_m is obtained:

$$T_m = \frac{1}{1-\left(\frac{\Gamma}{m\beta\lambda}\right)^2}\frac{\cos\left(\phi+\frac{\pi(d+g)}{\beta\lambda}\right)+\cos\left(\phi-\frac{\pi(l+g)}{\beta\lambda}\right)}{2\cos(\phi)}. \tag{14}$$

TTF has a specific physical meaning, which should be expressed by a continuous function. Mathematically, T_m has a singularity when the denominator in Eq. (14) equals to zero, and corresponding nominator at the point equals to zero as well. The T_m value can be obtained by solving the limit at the point.