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Analytic modeling, simulation and interpretation of broadband beam coupling impedance bench measurements

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ABSTRACT

First, a generalized theoretical approach towards beam coupling impedances and stretched-wire measurements is introduced. Applied to a circular symmetric setup, this approach allows to compare beam and wire impedances. The conversion formulas for TEM scattering parameters from measurements to impedances are thoroughly analyzed and compared to the analytical beam impedance solution. A proof of validity for the distributed impedance formula is given. The interaction of the beam or the TEM wave with dispersive material such as ferrite is discussed. The dependence of the obtained beam impedance on the relativistic velocity β is investigated and found as material property dependent.

Second, numerical simulations of wakefields and scattering parameters are compared. The applicability of scattering parameter conversion formulas for finite device length is investigated. Laboratory measurement results for a circularly symmetric test setup, i.e. a ferrite ring, are shown and compared to analytic and numeric models. The optimization of the measurement process and error reduction strategies are discussed.

1. Introduction

The concept of beam coupling impedance in circular accelerators was introduced by Vaccaro in 1966 to study coherent instabilities in the CERN ISR [1]. The determination of the impedance for the purpose of instability threshold or beam induced heat load estimation is nowadays mostly done by computer simulations. Nonetheless, for complicated accelerator devices such as kicker magnets, collimators, or diagnostic equipment one also requires laboratory measurement methods, such as the wire method.

The electromagnetic field distribution of a single particle in free space approaches the one of a lossless coaxial TEM transmission line in the ultrarelativistic limit [2]. This motivates measuring the longitudinal or transverse beam coupling impedance of accelerator components by replacing the beam with one or two wires, respectively. The transmission line measurement technique has been introduced by Sands and Rees [3] for the determination of beam energy loss factors in the Time Domain (TD) by a pulse excitation. When using modern Vector Network Analyzers (VNA) the beam coupling impedance can be determined in Frequency Domain (FD) by sweeping the frequency of a sinusoid, i.e. a narrow-band signal. Especially when looking at particular sidebands that are susceptible to beam instabilities rather than at the total energy loss the FD method is to be preferred.

In both TD and FD one has to make sure not to measure artifacts of the measurement setup, but rather the device as it is supposed to be placed in the beam line. The de-embedding process to measure only the Accelerator Device Under Test (DUT) was investigated especially for lumped impedances by Hahn and Pedersen [4]. In order to allow the de-embedding with a reference (REF) measurement of an empty box or beam pipe, the impedance mismatch from the cables to the measurement box must not exceed a certain value. Some techniques to achieve this are described by Kroyer et al. [5]. At high frequency one can also use 'Time Domain Gating' to disregard the mismatch reflections [6], but this requires a very high bandwidth of the VNA to properly represent the spectrum of the window-function. Another option is to damp multiple reflections with RF attenuation foam [7].

Walling et al. [8] first introduced an approximative formula for measuring distributed impedances. This was later replaced by the exact formula by Vaccaro [9]. Error considerations were performed by Hahn [10] and Jensen [11]. Hahn's paper gives explicit error estimates dependent on the electrical length derived from the integral equation also used by Gluckstern and Li [12]. Nonetheless, Hahn's expressions for distributed impedances are obtained by perturbation theory for small impedance, and they assume a priori a thin wire limit.

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Instead of using Gluckstern's integral equation, this paper approaches distributed impedances by computing the quasi-TEM-eigenmode, i.e. a TEM mode that has a small longitudinal field component which contains the losses. This mode's convergence to the fields for beam impedance calculation is shown for decreasing wire radius below higher order mode cutoff. This represents the other succession of the limits: the a priori assumption is infinite longitudinal electrical length and then the wire radius limit is investigated.

The paper generally covers analytical and numerical models for longitudinal and transverse impedance measurement of lossy broadband structures. The models will be applied to the test case of a dispersive ferrite ring which is treated analytically, numerically, and by measurement. Due to its dispersive character, the ferrite ring allows to benchmark the measurement interpretation formulas for different impedance ranges and different electrical lengths. Starting from a 2D analytical model, i.e. infinite electrical length, its limitations are illustrated by a 3D numerical model for finite length.

The interplay between measurements and simulations can be outlined as follows: on one hand, measurements serve to obtain impedances for $\beta=1$ which can validate simulations that allow scaling with β. On the other hand, numerical beam simulations for $\beta=1$ and scattering parameter (S-parameters see e.g. [13]) simulations are important to avoid wrong assumptions in the measurements.

The analytical model for the dispersive material presented here motivates also a simplified low frequency (LF) approach ("radial model" [14,15]) that plays an important role for the interpretation of LF impedance in general and in particular of coil measurements [16] for transverse impedance.

The paper is structured as follows: Section 2 starts with the analytical model for the beam impedance and for the measurement, i.e. a model with excitation and an eigenvalue problem, respectively. Both are solved for circularly symmetric 2D geometry. In Section 3 the way to determine the impedance from scattering parameters is discussed. The eigenmode approach from Section 2 is used to prove this relation for distributed impedances. Section 4 then draws an intermediate conclusion, comparing the 2D analytical beam and eigenvalue results only.

A real ferrite ring, as it has also been measured in practice, was simulated in 3D with a particle beam (TD) and a wire (TD/FD), as described in Section 5. An a posteriori justification of the 2D modeling in Section 2 is given here. Section 6 discusses the measurement process and its results. Technical details and material data error propagations are described in the appendices. The commonalities and differences for the longitudinal and transverse measurements are pointed out in Section 7. The paper concludes in Section 8 with a discussion of the different measurements methods and their respective simulation support.

2. Analytical model

From Maxwell's equations we find the 2D Helmholtz equation for the longitudinal electric field

$$(\Delta_\perp + k_\perp^2)E_z = \mathrm{rhs} \tag{1}$$

and the dispersion relation

$$k_\perp^2 + k_z^2 = \omega^2 \underline{\mu}\underline{\varepsilon} \tag{2}$$

where k_z and $k_\perp$ are the longitudinal and transverse wave numbers and $\underline{\mu}$ and $\underline{\varepsilon}$ are the complex permeability and permittivity as function of position and frequency $\omega = 2\pi f$, respectively. This will be solved for three different assumptions:

1. Beam model (β and γ are the relativistic velocity and mass factors)

$$k_z = \frac{\omega}{\beta c} \tag{3}$$

$$\mathrm{rhs} = -\frac{i\omega}{\beta^2\gamma^2}\mu_0 \frac{q}{\pi a^2} H(a-r) \tag{4}$$

with beam radius a, total charge q, and H being the Heaviside step function

2. Radial model obtained from beam model with $\beta \to \infty$, i.e.

$$k_z = 0, \quad \gamma = 0, \quad \beta\gamma = i, \quad \vec{E}_\perp = 0 \tag{5}$$

3. Coaxial line model (central wire radius a)

$$E_z(r \le a) = 0, \quad \text{quasi-TEM-eigenmode } (\mathrm{rhs}=0) \tag{6}$$

where k_z is an eigenvalue obtained from the equation

$$(\Delta_\perp + \omega^2 \underline{\mu}\underline{\varepsilon})E_z = k_z^2 E_z. \tag{7}$$

The range of validity of the radial model is also discussed in [14,15].

Before solving Eq. (1) we take a closer look on the dispersion relation Eq. (2), rewritten for the beam model as

$$k_\perp^2 = \frac{\omega^2}{c^2}\left(\underline{\mu}_r \underline{\varepsilon}_r - \frac{1}{\beta^2}\right). \tag{8}$$

The material properties are presented as

$$\underline{\mu} = \mu' - i\mu'' \quad \text{and} \quad \underline{\varepsilon} = \varepsilon' - i\varepsilon'' + \frac{\kappa}{i\omega} \tag{9}$$

with κ being the conductivity and μ'' and ε'' being magnetization and polarization losses, respectively. Furthermore we define the lossless refraction index and the loss tangents as

$$n = \sqrt{\mu_r' \varepsilon_r'}, \quad \tan\delta_\mu = \frac{\mu_r''}{\mu_r'} \quad \text{and} \quad \tan\delta_\varepsilon = \frac{\varepsilon_r'' + \kappa/\omega\varepsilon_0}{\varepsilon_r'}. \tag{10}$$

This allows to rewrite Eq. (8) as

$$k_\perp^2 = \frac{\omega^2}{c^2}\left[n^2(1-\tan\delta_\mu \tan\delta_\varepsilon) - \frac{1}{\beta^2} - in^2(\tan\delta_\mu + \tan\delta_\varepsilon)\right] \tag{11}$$

which shows that in the lossless case one has transversely propagating waves exactly when the Cherenkov-condition $\beta n > 1$

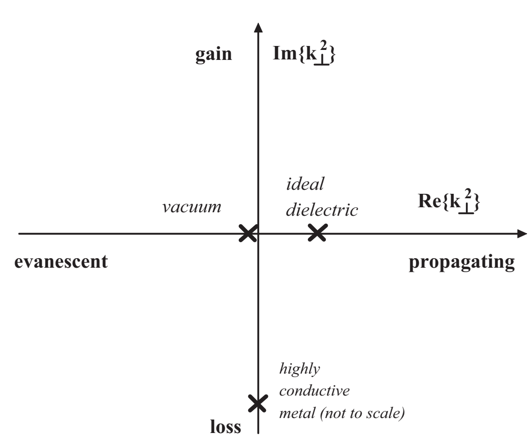

Fig. 1. Complex $k_\perp^2$ plane (transverse propagation plot). The vertical axis represents the Cherenkov-condition.