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Numerical investigation of the radiation characteristics of a variable-period helical undulator

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ABSTRACT

A helical undulator with a variable-period capability has been developed at the Korea Atomic Energy Research Institute (KAERI) to generate high power radiation in the terahertz range. A simulation code for the spontaneous emission from an electron beam inside an undulator has been developed to characterize the performance of the undulator. In the case of the KAERI undulator, there is a non-negligible high-order harmonics in the longitudinal field distribution compared with a bifilar one.The axial velocity modulation by the high-order harmonics in the field distribution has been found to lead to small deviation of the spectrum of spontaneous emission from the KAERI undulator with respect to the bifilars one. The gain functions obtained from the spontaneous emission spectra according to the Madey theory, show similar shapes for both undulators.

1. Introduction

Free electron lasers (FELs) [1] are famous for its wavelength tuning ability, which is usually realized by changing the magnetic field strength of an undulator through varying the gap distance of the undulator magnets or current in the case of an electromagnetic undulator. As an alternative way for wavelength tuning, a helical undulator with a variable-period capability has been developed at the Korea Atomic Energy Research Institute (KAERI) [2] to generate high power radiation at around 1 THz. Since the variation of the field strength is small as the period changes, the amplification gain can be kept almost constant in changing the lasing wavelength. The combination of the variable-period helical undulator and a compact microtron accelerator can realize a table-top FEL in the terahertz range.

The performance of the newly proposed KAERI undulator has been numerically investigated. Instead of a detailed simulation on the lasing dynamics, spontaneous emission spectra from the KAERI undulator with an electron beam from the microtron accelerator are compared with a bifilar one. There is a small discrepancy in the dynamics of an electron beam, which is caused by a high-order component in the field distribution of the KAERI undulator. The small-signal gain is almost the same for both undulators. The gain functions are also compared for the case when the central frequency of both undulators are set to same by changing the undulator period of the bifilar one, which also shows very similar shapes.

2. Numerical methods

A numerical code to investigate the performance of an undulator through a spontaneous emission spectrum is developed. The code consists of two parts, one for the evaluation of electron beam dynamics and the other for the generation of a spontaneous emission spectrum. In the electron beam dynamics part, the dynamics of each electron is evaluated by the relativistic Lorentz equation under an undulator magnetic field only. Then, the dynamics of every electron is fed into the spontaneous emission part to generate angular spectral intensities at a given direction using the Lienard–Wiechert potential, then all the radiations from electrons are incoherently added to produce the total spectrum.

2.1. Electron beam dynamics

Under the assumption that the space charge effect is negligible, the dynamics of each electron under an undulator magnetic field is evaluated by the following relativistic Lorentz equations written in

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Gaussian unit:

$$\gamma m_e c\frac{d\vec{u}}{dt} = -e\vec{u}\times\vec{B}, \tag{1}$$

where m_e and e are the electron mass and charge, respectively. c is the speed of light in vacuum. γ is the relativistic Lorentz factor, and $\vec{u}=\gamma\vec{v}$, where $\vec{v}$ is the velocity of an electron divided by c. The Boris method [3] is adopted to integrate above equation.

The initial electrons can be prepared either by listing up all the initial conditions of electrons or by sampling from a Gaussian distribution given the electron beam parameters. The Gaussian distribution for an electron beam in spatial position (x, y, z), transverse velocities normalized by the longitudinal velocity (x', y'), and energy (E) is expressed by the following formula:

$$\begin{aligned} f(x,x',y,y',z,E) \\ =\frac{1}{(2\pi)^3\sigma_E\sigma_z\epsilon_x\epsilon_y}\exp\Bigg[-\frac{\gamma_x x^2+2\alpha_x xx'+\beta_x x'^2}{2\epsilon_x} \\ -\frac{\gamma_y y^2+2\alpha_y yy'+\beta_y y'^2}{2\epsilon_y}-\frac{(E-E_o)^2}{2\sigma_E^2}-\frac{(z-z_o)^2}{2\sigma_z^2}\Bigg], \end{aligned} \tag{2}$$

where $\alpha_{x,y}$, $\beta_{x,y}$, and $\gamma_{x,y}$ are Twiss parameters in the x and y directions and satisfy $\gamma_{x,y}=(1+\alpha_{x,y}^2)/\beta_{x,y}$ [4]. E_o is the nominal beam energy, σ_E is the energy spread, σ_z is the longitudinal beam length, and $\epsilon_{x,y}$ are the transverse emittances. Twiss parameters and emittances are related with the transverse beam size $\sigma_{x,y}$ and the beam divergence $\theta_{x,y}$ by the following relations:

$$\gamma_{x,y}=\theta_{x,y}^2/\epsilon_{x,y}, \tag{3}$$

$$\beta_{x,y}=\sigma_{x,y}^2/\epsilon_{x,y}. \tag{4}$$

The beam parameter can be defined either by the Twiss parameters, or by the beam sizes and the divergences in the transverse directions.

2.2. Spontaneous emission

Once the dynamics of an electron beam is obtained, the angular spectral intensity of the spontaneous emission from each electron toward the direction $\hat{n}$ is calculated by the followings [5]:

$$\frac{d^2I}{d\omega d\Omega}=2\left|\vec{A}(\omega)\right|^2, \tag{5}$$

$$\vec{A}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty}\vec{A}(t)e^{-i\omega t}dt, \tag{6}$$

$$\vec{A}(t)=\sqrt{\frac{e^2}{4\pi c}}\left[\frac{\hat{n}\times\{(\hat{n}-\vec{v})\times\dot{\vec{v}}\}}{(1-\hat{n}\cdot\vec{v})^3}\right]_{t'}, \tag{7}$$

where $\dot{\vec{v}}$ is the time derivative of $\vec{v}$ and t' is the electron's time or retarded time, which is related to t by

$$t=t'+\frac{x-\hat{n}\cdot\vec{r}(t')}{c}, \tag{8}$$

where x is the distance from the origin to a detector under the condition of x ÃAA aA $|\vec{r}|$. All the radiations from the electrons in an electron beam are then incoherently added in the angular frequency space.

3. Results and discussions

For a comparative study, the dynamics of an electron beam in the KAERI undulator is compared with a bifilar one. The first harmonic magnetic field of the bifilar undulator is expressed in the cylindrical coordinate as follows [6]:

$$\vec{B}=2B_u f(z)\left[\hat{e}_r I_1'(k_u r)\cos\chi-\hat{e}_\phi\frac{1}{k_u r}I_1(k_u r)\sin\chi+\hat{e}_z I_1(k_u r)\sin\chi\right], \tag{9}$$

where I_1 is the modified bessel function of order one and $\chi=\phi-k_u z$. I_1' is the derivative of the I_1 by $k_u r$. B_u is the amplitude of the undulator field and $k_u=2\pi/\lambda_u$ with the period of the undulator λ_u. The adiabatic field at the entrance and the exit sections of the undulator is expressed by $f(z)$, which can be formulated as follows:

$$f(z)=\begin{cases}\sin^2\left(\frac{\pi}{2}\frac{z}{\Delta z_i}\right), & 0\le z<\Delta z_i\\ 1, & \Delta z_i\le z<N_u\lambda_u-\Delta z_f\\ \sin^2\left(\frac{\pi}{2}\frac{(z-N_u\lambda_u)}{\Delta z_f}\right), & N_u\lambda_u-\Delta z_f\le z<N_u\lambda_u\\ 0, & \text{otherwise}\end{cases} \tag{10}$$

where Δz_i and Δz_f are the length of adiabatic sections at the entrance and the exit sections, respectively. N_u is the total number of periods including adiabatic sections.

At KAERI, a variable-period helical undulator has been developed [2]. As shown in Fig. 1, the basic module is composed of two iron poles and two permanent magnets embedded in a non-magnetic plate. The magnetizations of the two magnets in the module are in opposite directions $\pm z$. The base plates are arranged with a 90° rotation in series, thus four plates complete one period of a helical magnetic field distribution. Since the base plates are balanced by a repulsive force next to adjacent plates, the undulator period can be adjusted by changing the position of an end plate [2], which makes it possible to adjust the period between 2.3 cm and 2.6 cm.

For the case of $\lambda_u=2.3$ cm, magnetic field distributions are compared between the bifilar and KAERI undulators in Fig. 2. In the case of the KAERI undulator, the field distribution was numerically obtained by the CST code [2]. The comparison of the undulators was made at the same value of the peak field B_u. One can see that there are additional high-order oscillations over a 2.3 cm oscillation in B_x of the KAERI undulator (Fig. 2(a)). It has been found that the next higher-order component to the fundamental one is the 3rd order by the Fourier transformation. The field distributions of the KAERI undulator on the axis can be described as follows:

$$B_x=A_1\cos(k_u z)+A_3\cos(3k_u z), \tag{11}$$

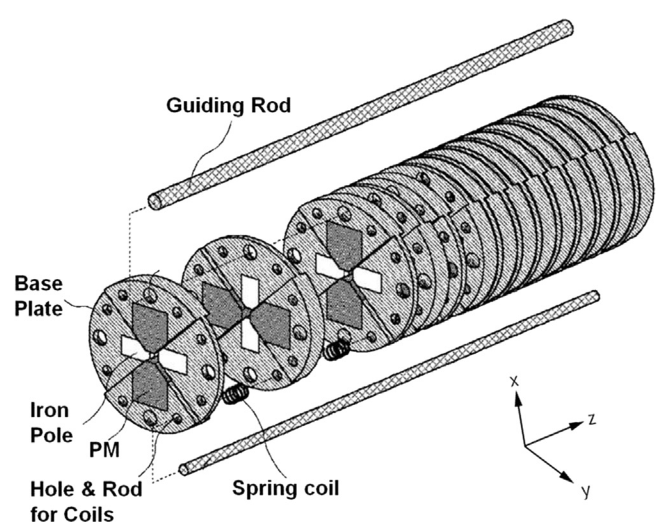

Fig. 1. Schematic of a variable-period helical undulator developed at KAERI. Each base plate is composed of two iron poles and two permanent magnets embedded in a non-magnetic plate. Each plate is arranged in series with a 90° rotation to make a helical field distribution.