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Consideration of spatial variation of the length scale parameter in static and dynamic analyses of functionally graded annular and circular micro-plates

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ABSTRACT

This article introduces new methods for static and free vibration analyses of functionally graded annular and circular micro-plates, which can take into account spatial variation of the length scale parameter. The underlying higher order continuum theory behind the proposed approaches is the modified couple stress theory. A unified way of expressing the displacement field is adopted so as to produce numerical results for three different plate theories, which are Kirchhoff plate theory (KPT), Mindlin plate theory (MPT), and third-order shear deformation theory (TSDT). Governing partial differential equations and corresponding boundary conditions are obtained following the variational approach and the Hamilton's principle. Derived systems of differential equations are solved numerically by utilizing the differential quadrature method (DQM). Comparisons to the results available in the literature demonstrate the high level of accuracy of the numerical results generated through the developed methods. Extensive analyses are presented in order to illustrate the influences of various geometric and material parameters upon static deformation profiles, stresses, and natural vibration frequencies. In particular, the length scale parameter ratio -which defines the length scale parameter variation profile-is shown to possess a profound impact on both static and dynamic behaviors of functionally graded annular and circular micro-plates.

1. Introduction

Functionally graded materials (FGMs) belong to a special class of composites, which possess smooth spatial variations in the volume fractions of the constituent phases. These variations facilitate design of structures with customized physical properties and improved performance. Previous work show that property gradation enhances mechanical response of components utilized in a number of technological applications including thermal barrier coatings [1], wear resistant surfaces [2], cutting tools [3], solid oxide fuel cells [4], and biomaterials [5]. Recently, with the advent of micro-scale FGM component fabrication techniques such as magnetron sputtering [6], chemical vapor deposition and plasma-enhanced chemical vapor deposition [7], and modified soft lithography [8]; research focus has been placed on behavior of functionally graded micro-scale structures.

The present study aims at putting forward new analysis techniques for micro-scale annular and circular plates built from functionally graded materials. Annular and circular micro-plates have been employed as components in a wide variety of micro-electro-mechanical-systems (MEMS). Annular micro-plates are utilized in MEMS such as gear pumps [9], stiction valves [10], and resonators [11,12]; whereas circular micro-plates find applications in pressure sensors [13], acoustic energy harvesters [14], and optical MEMS sensors [15]. Functionally graded annular and circular plates possess the intrinsic advantages that come along with spatial variations in physical properties. Depending on the form of these variations, in graded annular and circular plates deflections and tensile static stresses could be lower [16] and critical buckling loads could be higher [17] compared to those evaluated for non-graded counterparts. Furthermore, natural frequencies of free vibrations of a graded plate are strongly dependent upon property distribution profiles and can be optimized by devising a suitable composition architecture [18]. Thus, it is of significance to employ solution methodologies delivering sufficiently accurate results regarding

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both static and dynamic behaviors of graded annular and circular micro-plates.

Conventional continuum theories, such as classical elasticity, can not accurately predict the response of micro-components because of the size effect prevailing at the micro-scale. Analysis of micro-scale structures needs to be based on a higher order continuum theory, examples of which are nonlocal theory of Eringen [19–21], modified couple stress theory [22–24], strain gradient elasticity [25,26], and finite deformation gradient elasticity [27]. Modified couple stress theory and strain gradient elasticity are the most commonly used higher-order theories in the analyses of micro-scale functionally graded components.

There are several articles in the technical literature that present higher order continuum theory based analysis techniques for micro-scale FGM plates. Adopting modified couple stress theory, Kim and Reddy [28] and Thai and Kim [29] developed solutions for rectangular graded micro-plates; and Asghari and Taati [30] treated the arbitrarily shaped micro-plate problem. Micro-scale axisymmetric functionally graded plates were considered by Ansari et al. [17] and Ke et al. [18]. Ansari et al. [17] studied bending, buckling, and free vibrations of annular and circular micro-plates using modified strain gradient elasticity, whereas Ke et al. [18] examined free vibrations of annular plates by employing modified couple stress theory.

Higher order stresses and strain gradient measures in higher order continuum theories are related through length scale parameters. The single length scale parameter in modified couple stress theory for instance is defined as the ratio of the modulus of curvature to the shear modulus [31,32], and thus within this context it is essentially an elastic material property. Since all elastic properties of a functionally graded structure are expected to possess spatial variations, the length scale parameter is in general also a function of the spatial coordinates. Hence, the employed solution methodology should be able to account for the spatial variation of the length scale parameter. However, in all studies mentioned in the foregoing paragraph, length scale parameter is assumed to be a constant quantity. The only study presenting a systematic approach in the consideration of the spatial variation of the length scale parameter is that by Aghazadeh et al. [33], which treats problems involving functionally graded micro-beams. Yet, spatial variation of the length scale parameter has not been incorporated into the analysis of FGM micro-plates.

The main objective of the present study is to put forth modified couple stress theory-based modeling and analysis techniques for functionally graded annular and circular micro-plates, which take into account the *spatial variation of the length scale parameter*. Both static bending and free vibration problems of FGM micro-plates are studied to develop the proposed methods. In the formulation, displacement field is expressed in a certain unified form so as to generate results for three different plate theories, which are Kirchhoff plate theory (KPT), Mindlin plate theory (MPT), and third-order shear deformation theory (TSDT). Governing partial differential equations and boundary conditions are derived by applying Hamilton's principle in accordance with modified couple stress theory. All material properties including the length scale parameter are assumed to be functions of the thickness coordinate in the derivations. The equations are solved numerically by means of the differential quadrature method (DQM). Comparisons of the generated numerical results to those available in the literature illustrate the high degree of accuracy attained by the application of the developed procedures. Further parametric analyses are carried out to shed light upon the influences of material and geometric parameters on static deflections and natural frequencies of annular and circular FGM micro-plates. Numerical results unequivocally demonstrate that in modeling and analysis of graded micro-structures, it is necessary to take into consideration the spatial variation of the length scale parameter.

2. Formulation

In this study, we examine static and dynamic behaviors of functionally graded annular and circular micro-plates. The geometry of the annular micro-plate is depicted in Fig. 1. Inner and outer radii are respectively denoted by R_i and R_o. Circular micro-plate has an identical geometry except for the fact that $R_i = 0$. The loading function and the boundary conditions are assumed to be independent of θ, hence the underlying problems become axisymmetric. Material properties vary continuously along the z-direction. Both the annular and the circular micro-plates are 100% metallic at $z = -h/2$ and 100% ceramic at $z = h/2$. According to the modified couple stress theory [22], strain energy of the plates under consideration reads:

$$U = \frac{1}{2}\iiint_V (\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij})dV, \tag{1}$$

where V designates volume; σ_{ij} represents Cauchy stress; ε_{ij} is strain; m_{ij} stands for the deviatoric part of the couple stress tensor; and χ_{ij} is the symmetric curvature tensor. The tensorial quantities are expressed in the following form:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk}, \tag{2a}$$

$$m_{ij} = 2\mu l^2\chi_{ij}, \tag{2b}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}\right], \tag{2c}$$

$$\boldsymbol{\chi} = \frac{1}{2}\left[\nabla\boldsymbol{\omega} + (\nabla\boldsymbol{\omega})^{\mathrm{T}}\right]. \tag{2d}$$

In Eq. (2), μ and λ are Lame parameters, δ_{ij} is Kronecker delta, l denotes the length scale parameter, $\mathbf{u}$ symbolizes displacement vector, and $\boldsymbol{\omega}$ is the rotation vector. μ, λ, and the rotation vector are defined by

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \tag{3a}$$

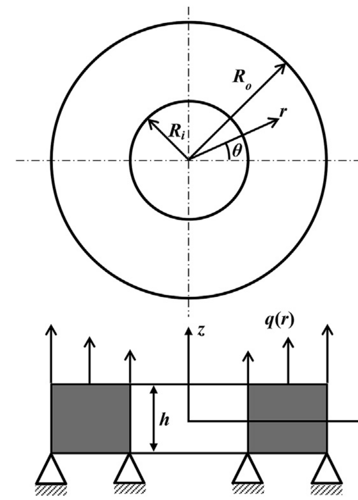

Fig. 1. A functionally graded annular micro-plate under the effect of distributed loading $q(r)$.