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Fourier-based reconstruction via alternating direction total variation minimization in linear scan CT

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ABSTRACT

In this study, we consider a novel form of computed tomography (CT), that is, linear scan CT (LCT), which applies a straight line trajectory. Furthermore, an iterative algorithm is proposed for pseudo-polar Fourier reconstruction through total variation minimization (PPF-TVM). Considering that the sampled Fourier data are distributed in pseudo-polar coordinates, the reconstruction model minimizes the TV of the image subject to the constraint that the estimated 2D Fourier data for the image are consistent with the 1D Fourier transform of the projection data. PPF-TVM employs the alternating direction method (ADM) to develop a robust and efficient iteration scheme, which ensures stable convergence provided that appropriate parameter values are given. In the ADM scheme, PPF-TVM applies the pseudo-polar fast Fourier transform and its adjoint to iterate back and forth between the image and frequency domains. Thus, there is no interpolation in the Fourier domain, which makes the algorithm both fast and accurate. PPF-TVM is particularly useful for limited angle reconstruction in LCT and it appears to be robust against artifacts. The PPF-TVM algorithm was tested with the FORBILD head phantom and real data in comparisons with state-of-the-art algorithms. Simulation studies and real data verification suggest that PPF-TVM can reconstruct higher accuracy images with lower time consumption.

1. Introduction

The recently developed linear scanning computed tomography (LCT) method with a straight line trajectory [1–8] has promising applications in medical imaging and industrial or security inspections because of its high scanning efficiency. However, because it is limited by ray-beam flare angle and the detector size, exact image reconstruction with LCT is challenging due to the severe violation of the Tuy-Smith sufficient condition [9,10]. Thus, the reconstruction procedure can be treated as an ill-posed inverse problem, which generally has no unique solution due to the lack of sufficient measurements and the presence of noise in the data. Conventional reconstruction algorithms [1–6,11–13] such as the filtered back-projection (FBP) algorithms, the back-projection filtration algorithms, the direct Fourier methods (DFM), the algebraic reconstruction technique (ART), and the simultaneous algebraic reconstruction technique (SART) cannot provide images with satisfactory quality, which has been verified based on theoretical and experimental results.

Compressive sensing (CS) [14–16] exploits the sparsity or compressibility of the signal and it reconstructs the signal exactly using optimization-based methods with fewer incoherent linear projections than conventional approaches. It all cases, the massive images used in medical and industrial fields possess very sparse gradient magnitude images, thereby facilitating the development of CS-based reconstruction methods. Consequently, total variation (TV) minimization has been applied successfully to CT image reconstruction from sparse or limited view datasets [17–19]. The adaptive steepest descent-projection onto convex sets (ASD-POCS) [19] was developed to provide much better reconstructions than many previous algorithms, but it requires intensive computation and satisfactory reconstructions may need numerous iterations. Several variants [20,21] have been proposed based on the ASD-POCS framework for different applications, where the scanning process and data acquisition methods were tailored specifically.

Based on the constrained TV and l_1 optimization model, the recently developed augmented Lagrange-based alternating direction method of multipliers (ADMM) and split Bregman iteration have high efficiency and solid convergence in CS applications [22–24]. It must be noted that the two methods are equivalent with linear observations [25,26] and both have been applied to CT [27–29] and magnetic resonance imaging [30,31]. The reconstruction from partial Fourier (RecPF) data [31], an algorithm that uses ADMM for Fourier data reconstruction, which was proposed by Yang, has been shown to be both efficient and accurate. However, RecPF is designed for a

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Cartesian distribution with sampling in both the Fourier and image domains, which means that it is not suitable for CT reconstruction. A variant of RecPF that applies sparse resampling interpolation (RecPF-SR) [32] has been proposed for LCT imaging, which yielded some impressive results.

Among the different CT reconstruction algorithms, it is known that Fourier-based reconstruction algorithms [33–36] are more efficient than others due to the application of fast Fourier transform (FFT) techniques. However, the DFM has lower reconstruction accuracy due to interpolation in the frequency domain. Because the sampling frequency of LCT projection follows a pseudo-polar distribution where the transform can be executed by pseudo-polar FFT (PPFFT) [37] without interpolation, the reconstruction from projections can be converted into reconstruction from pseudo-poplar distributed Fourier sampling.

PPFFT is mathematically and algebraically exact, geometrically faithful and invertible, and it has already been used in tomography [38–42]. In the present study, we describe a fast and accurate LCT reconstruction algorithm based on TV minimization and PPFFT with the alternating direction method (ADM) [43,44] framework. The proposed algorithm minimizes the TV norm of the image subject to the constraint that the estimated 2D Fourier data of the image is exactly consistent with the 1D Fourier transform of the projection data. To handle this constrained optimization, the ADM is utilized to solve it efficiently, thereby making the proposed algorithm, that is, pseudo-polar Fourier reconstruction based on TV minimization (PPF-TVM), both fast and accurate.

The remainder of this study is organized as follows. Section 1 discusses reconstruction issues and the development of the appropriate algorithms for LCT. Section 2 presents our reconstruction model and the corresponding PPF-TVM algorithm. In Section 3, the numerical simulations and real data experiments are used to validate the reconstruction quality of the PPF-TVM algorithm. In Section 4, we provide a brief discussion and we give our conclusions in Section 5.

2. Method

2.1. Systematic description and data model

A linear scan CT considered in this article is shown in Fig. 1(a). The configuration can be described as follows. In a complete scanning, the object is transferred from the left to the right through the radiation area moving in a straight line trajectory while keeping the X-ray source and the detector stationary.

The projection $p(l,t)$ [4] from an object $f(x,y)$ in a continuous form can be expressed as

$$\begin{aligned} p(l,t) &= \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy f(x,y)\delta(x\cos\theta + y\sin\theta - s) \\ &= \frac{\sqrt{D^2+t^2}}{D} \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy f(x,y)\delta\left(x - y\frac{t}{D} - l - t\right), \end{aligned} \tag{1}$$

where $\theta = \pi - \tan^{-1}(t/D)$ and $s = -D(l+t)/\sqrt{D^2+t^2}$. From Eq. (1), if we fix parameter t as a constant, then Eq. (1) is equivalent to a parallel beams projection under viewing angle θ, where s denotes the corresponding detector.

Preprocessed data $q(l,t) = p(l-t,t)$ [5] can be useful for saving memory and reducing computations. Thus, if we take the Fourier transform of $p(l,t)$ with respect to variant l we obtain

$$\begin{aligned} \hat{q}(\omega,t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dl q(l,t) e^{-j\omega l} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dl p(l-t,t) e^{-j\omega l} \\ &= \frac{1}{\sqrt{2\pi}} \frac{\sqrt{D^2+t^2}}{D} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx dy f(x,y) e^{-j\omega\left(x - y\frac{t}{D}\right)} \\ &= \sqrt{2\pi} \frac{\sqrt{D^2+t^2}}{D} \hat{f}(\omega, -\omega t/D). \end{aligned} \tag{2}$$

This means that

$$\hat{f}(\omega, -\omega t/D) = \frac{D/\sqrt{2\pi}}{\sqrt{D^2+t^2}} \hat{q}(\omega,t), \tag{3}$$

where $\hat{f}$ is the 2-D Fourier transform of object function f. During LCT scanning, the Fourier transform of the projection data acquired by a certain detector element t gives a Fourier slice of the object function f in the Fourier domain, which is called the "Fourier slice theorem" for an LCT scan [5]. For linearly spaced l and t, the sampling points in the Fourier domain $\hat{f}(\omega_1,\omega_2)$ are linearly spaced in the ω_1 direction whereas they are nonlinear in the ω_2 direction, and all are distributed on an equally sloped straight line that passes through the origin, which is called the pseudo-polar distribution [15]. The transform from the image domain to the 2D frequency domain can be executed by PPFFT without interpolation [37].

In order to understand this issue better, Fig. 2 shows the frequency distribution. Clearly, when the detector length is ideally infinite and the source flare angle ϕ equals π, the Fourier transform of projection q collects the complete Fourier data of a 2-D image f. A natural reconstruction method, that is, the DFM, for reconstructing f performs the inverse Fourier transform:

$$f(x,y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\sigma |\omega_1| \hat{f}(\omega_1, \omega_1\sigma) e^{j(\omega_1 x + \omega_1 \sigma y)}, \tag{4}$$

where $(\sigma = -\frac{t}{D})$ indicates the slope of the central Fourier slice. A typical method for direct reconstruction from samples (Fig. 2) is known as a linogram [6,34,35], which is based on FFT techniques. However, in practical applications, the detector cannot be of infinite length and the source angle ϕ is less than π. Therefore, $\hat{f}(\omega_1,\omega_1\sigma)$ only collects partial data in the Fourier domain and the integral limits for variant σ in Eq. (4) lie within $[-\tan\phi/2, +\tan\phi/2]$ instead of $(-\infty,+\infty)$; thus, the reconstruction from Eq. (4) is simply an approximation of f with little practical use.

2.2. Reconstruction model and algorithm

LCT reconstruction is a type of inverse problem, that is, we need to convert observed measurements of projections into images that reflect a physical object of interest. From the perspective of DFM, the equivalent observation in the Fourier domain is as follows:

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx dy f(x,y) e^{-j(\omega_1 x + \sigma\omega_1 y)} = \hat{f}(\omega_1, \omega_1\sigma), \tag{5}$$

where $\sigma = -\frac{t}{D}$ and $\hat{f}(\omega_1,\omega_1\sigma)$ is observed by the Fourier transform of the measured projection data $q(l,t)$ in (3), provided that there are $2M$ detector elements (indexed by m) with an interval $\triangle t$ and the projection data at $2N$ source positions (indexed by n) are collected with interval $\triangle l$. Consequently, ω_1 and σ can be discretized as $\frac{n\pi}{\triangle lN}$ and $\frac{m\triangle t}{D}$, respectively. Thus, the discrete form of Eq. (5) can be written as the following discrete Fourier transform:

$$\frac{\triangle l^2}{2\pi} \sum_{k_1=-N}^{N-1} \sum_{k_2=-N}^{N-1} f(k_1\triangle l, k_2\triangle l) e^{-j\left(\frac{n\pi}{\triangle lN}k_1\triangle l + \frac{n\pi}{\triangle lN}\cdot\frac{m\triangle t}{D}k_2\triangle l\right)} = \hat{f}\left(\frac{n\pi}{\triangle lN}, \frac{n\pi}{\triangle lN}\cdot\frac{m\triangle t}{D}\right), \tag{6}$$

where $f(k_1\triangle l, k_2\triangle l)$ denotes the unknown object function in discrete form. The observed data $\hat{f}(n,nm)$ comprise a linear combination of $f(k_1,k_2)$ with complex coefficients. The image reconstruction task aims to find the solution of Eq. (6) and thus for clarity of expression we rewrite Eq. (6) as the following discrete linear system:

$$F_p \vec{f} = \hat{f}. \tag{7}$$