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Dynamics of intense particle beam in axial-symmetric magnetic field

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ABSTRACT

Axial-symmetric magnetic field is often used in focusing of particle beams. Most existing ion Low Energy Beam Transport lines are based on solenoid focusing. Modern accelerator projects utilize superconducting solenoids in combination with superconducting accelerating cavities for acceleration of high-intensity particle beams. Present article discusses conditions for matched beam in axial-symmetric magnetic field. Analysis allows us to minimize power consumption of solenoids and beam emittance growth due to nonlinear space charge, lens aberrations, and maximize acceptance of the channel. Expressions for maximum beam current in focusing structure, beam emittance growth due to spherical aberrations and non-linear space charge forces are derived.

1. Lattice of periodic solenoid channel

Consider a focusing lattice consisting of a periodic sequence of focusing solenoids of length D, field B_o, distance between lenses l, and period $L = l + D$ (see Fig. 1). A matched beam reaches its maximum size in the center of the solenoids, and minimum size in the middle of drift space (see Fig. 2). The transformation matrix in a rotating frame through a period of the structure between centers of solenoids is given by [1]

$$\begin{pmatrix} \cos\frac{\theta}{2} & \frac{D}{\theta}\sin\frac{\theta}{2} \\ -\frac{\theta}{D}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & \frac{D}{\theta}\sin\frac{\theta}{2} \\ -\frac{\theta}{D}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta - \frac{l}{2D}\theta\sin\theta & \frac{D}{\theta}\sin\theta + l\cos^2\frac{\theta}{2} \\ -\frac{\theta}{D}\sin\theta + l\left(\frac{\theta}{D}\right)^2\sin^2\frac{\theta}{2} & \cos\theta - \frac{l}{2D}\theta\sin\theta \end{pmatrix}, \quad (1.1)$$

where θ is the rotational angle of particle trajectory in a solenoid:

$$\theta = \frac{qB_oD}{2mc\beta\gamma}. \quad (1.2)$$

The matrix of transformation through the period of the structure between centers of drift space is:

$$\begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \frac{D}{\theta}\sin\theta \\ -\frac{\theta}{D}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta - \frac{l}{2D}\theta\sin\theta & l\cos\theta - \frac{l^2\theta}{4D}\sin\theta + D\frac{\sin\theta}{\theta} \\ -\frac{\theta}{D}\sin\theta & \cos\theta - \frac{l}{2D}\theta\sin\theta \end{pmatrix} \quad (1.3)$$

From the matrices, Eqs. (1.1) and (1.3), the value of betatron tune shift per period, μ_o, is determined by

$$\cos\mu_o = \cos\theta - \theta\sin\theta\frac{(L-D)}{2D}. \quad (1.4)$$

Adopting the expansions $\cos\xi = 1 - \xi^2/2 + \xi^4/24$ and $\sin\xi = \xi - \xi^3/6$, the value of betatron tune shift per period reads:

$$\mu_o = \theta\sqrt{\frac{L}{D}}\sqrt{1 - \frac{\theta^2}{6}\left[1 - \frac{1}{2}\left(\frac{D}{L} + \frac{L}{D}\right)\right]}. \quad (1.5)$$

Thus, the maximum and minimum values of the beta-function $\beta_{max/min} = m_{12}/\sin\mu_o$ in the channel are given by:

$$\beta_{max} = \frac{L\cos^2\frac{\theta}{2}\left[1 - \frac{D}{L}\left(1 - \frac{\tan\theta/2}{(\theta/2)}\right)\right]}{\sin\mu_o}, \quad (1.6)$$

$$\beta_{min} = \frac{(L-D)\cos\theta - \frac{(L-D)^2\theta}{4D}\sin\theta + D\frac{\sin\theta}{\theta}}{\sin\mu_o}. \quad (1.7)$$

Eqs. (1.6), (1.7) determine the maximum $R_{max} = \sqrt{\beta_{max} \ni}$ and minimum $R_{min} = \sqrt{\beta_{min} \ni}$ matched envelope of the beam with unnormalized emittance, $\ni$, and negligible beam current, $I = 0$. Acceptance of the channel with aperture radius, a, is given by

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$A = a^2/\beta_{max}$:

$$A = \frac{a^2 \sin \mu_o}{L\left[1 - \frac{D}{L}\left(1 - \frac{\tan \theta/2}{(\theta/2)}\right)\right] \cos^2\frac{\theta}{2}}. \quad (1.8)$$

The acceptance, Eq. (1.8), is maximized at a betatron tune shift within the range $0 < \mu_o < 180^o$ (see Fig. 3b, solid line).

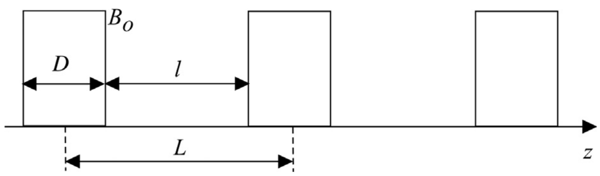

Fig. 1. Periodic structure of focusing solenoids.

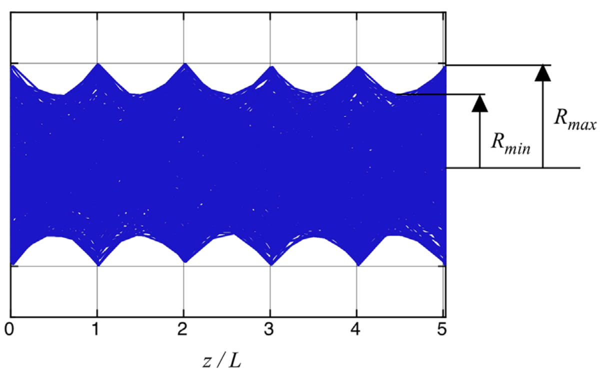

Fig. 2. A matched beam in a periodic focusing structure.

2. Thin lens approximation

If the thickness of the lens is significantly smaller than the period of the structure, D/L≪1, focusing properties of the solenoid can be represented by the thin lens matrix with focal length f:

$$M = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \quad f = \frac{D}{\theta^2}. \quad (2.1)$$

Consequently, the betatron tune shift per period of the structure is determined from Eq. (1.4) as:

$$\cos \mu_o = 1 - \theta^2 \frac{L}{2D} = 1 - \frac{L}{2f}, \quad (2.2)$$

from which the value of μ_o is

$$\mu_o \approx \theta\sqrt{\frac{L}{D}} = \sqrt{\frac{L}{f}}. \quad (2.3)$$

From the condition $|\cos \mu_o| \le 1$, the stability criteria for particle oscillations is expressed as $0 \le L \le 4f$ [2]. Accordingly, the acceptance of the channel is simplified from Eq. (1.8) as

$$A \approx \frac{a^2}{L} \sin \mu_o \quad (2.4)$$

and has a maximum at the value of $\mu_o \approx \pi/2$. In this case, $\cos \mu_o = 1 - L/2f = 0$, and $f = L/2$ which expresses a condition of symmetry of the channel.

The values of beta-function from Eqs. (1.6) and (1.7) are approximated as

$$\beta_{min} = \frac{L}{\sin \mu_o}\left(1 - \frac{\mu_o^2}{4}\right), \quad \beta_{max} = \frac{L}{\sin \mu_o}. \quad (2.5)$$

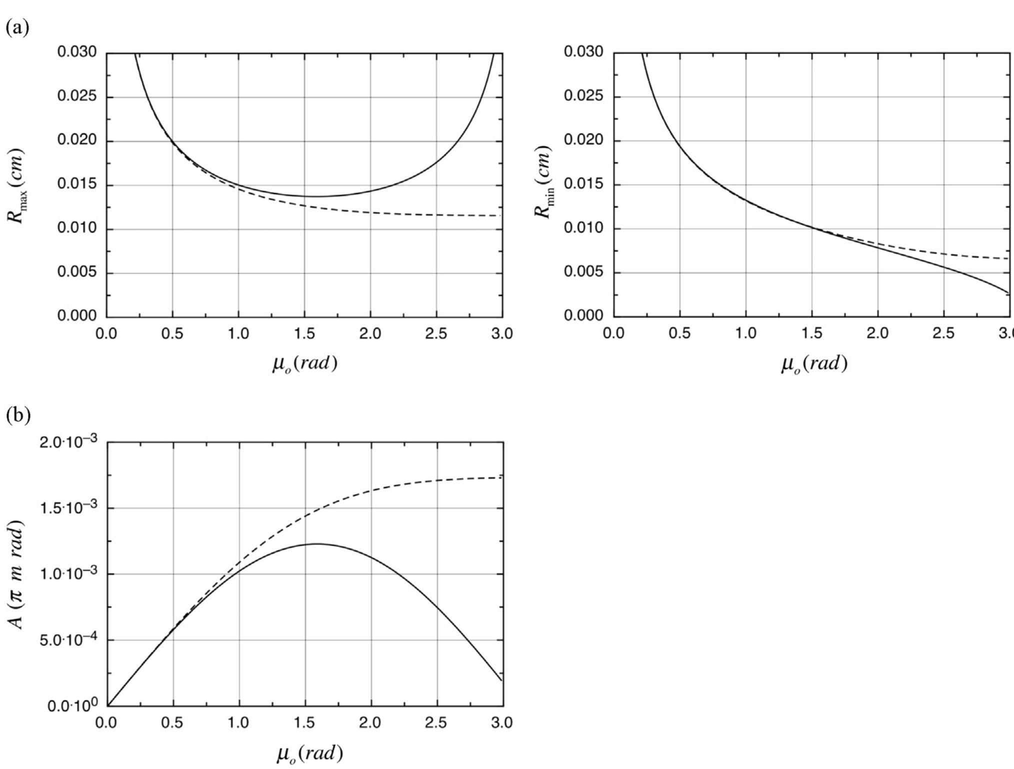

Fig. 3. (a) Minimum and maximum beam sizes in a periodic solenoid structure with D/L=0.034: (solid line) solution from matrix analysis, Eqs. (1.6) and (1.7), (dotted line) smooth approximation to the beam envelope, Eq. (3.26), (b) acceptance of the channel: (solid line) determined by matrix method, Eq. (1.8), (dotted line) determined from envelope equation, Eq. (3.29).