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Three-dimensional free and transient vibration analysis of composite laminated and sandwich rectangular parallelepipeds: Beams, plates and solids

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ABSTRACT

This paper presents an efficient method for predicting the free and transient vibrations of multilayered composite structures with parallelepiped shapes including beams, plates and solids. The exact three-dimensional elasticity theory combined with a multilevel partitioning hierarchy, viz., multilayered parallelepiped, individual layer and layer segment, is employed in the analysis. The continuity constraints on common interfaces of adjacent layer segments are imposed by a modified variational principle, and the displacement components of each layer segment are assumed in the form of orthogonal polynomials and/or trigonometric functions. Numerical studies are given for free vibrations of composite laminated and sandwich beams, plates, and solids. Some in-plane shear vibration modes missed in previous elasticity solutions for multilayered plates are examined. The natural frequencies derived from Reddy's high-order shear deformation theory and layerwise theory for soft-core sandwich plates show significant deviation from elasticity solutions. Transient displacement and stress responses for angle-ply laminated and sandwich plates under four types of impulsive loads (including rectangular, triangular, half-sine and exponential pulses) are obtained by the Newmark integration procedure. The present solutions may serve as benchmark data for assessing the accuracy of advanced structural theories and new developments in numerical methods.

1. Introduction

Composite laminated and sandwich structures are widely used in defence and aerospace industries due to their excellent mechanical properties including high values of stiffness-to-weight and strength-to-weight ratios. Since defence and aerospace applications allow small design margins, an exact knowledge of the dynamic behaviors of composite structures is of particular importance to structural analysts. As a consequence, a tremendous interest in the analyses of multilayered composite structures has emerged in the last few decades. In particular, the determination of dynamic behaviors of multilayered rectangular parallelepipeds is a fundamental topic since a variety of practical structures including beams, plates, and solids may be represented by parallelepipeds with different geometrical parameters.

Three-dimensional (3-D) dynamic analyses of multilayered rectangular parallelepipeds have received very little treatment in the previously published literature despite the fact that the problems are well defined and have been understood for well over a long time. In particular, if one dimension of a rectangular parallelepiped is very large compared to the other two, the parallelepiped will act as a long beam, whereas a parallelepiped becomes a plate when one of the dimensions is much smaller compared to the remaining two. If a parallelepiped does not fall into one of these categories, it may be physically viewed as a solid structure. Note that the classification of beams, plates, and solids made here is merely to represent the general characteristics of the geometry rather than strict definitions. Regarding the 3-D free vibration problems of multilayered rectangular parallelepipeds, exact solutions in terms of trigonometric functions can be obtained for limited cases; for example, a rectangular parallelepiped is cross-ply laminated and under simply-supported boundary conditions. Consequently, most of the approaches for the vibration problems of multilayered parallelepipeds are approximate in nature. Using the trigonometric function method, Srinivas et al. [1,2] obtained the natural frequencies of simply-supported orthotropic rectangular plates and laminates. Noor and Burton [3,4] presented analytical 3-D elasticity solutions for the free vibration problems of multilayered anisotropic plates, which were assumed to have

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rectangular geometry and antisymmetric lamination with respect to the middle plane. Ye and Soldatos [5] proposed a recursive solution for free vibration analysis of simply-supported, cross-ply laminated plates with an arbitrary number of point supports. Later, the recursive solution was extended by Ye [6] to the free vibration problems of cross-ply laminated rectangular plates with clamped boundaries. Cheung and Chakrabarti [7] developed a finite layer method for analyzing the free vibrations of thick, layered rectangular plates with various boundary conditions. This method was further extended by Zhou et al. [8] to the vibration analyses of multilayered rectangular plates with point supports. Chen and Lü [9] examined the free vibrations of cross-ply laminated composite rectangular plates, for which one pair of opposite edges is assumed to be simply-supported. Lü et al. [10] carried out a free vibration analysis for composite laminated plates based on 3-D elasticity theory using the state-space approach combined with the differential quadrature method. Zhang et al. [11] performed a free vibration analysis for simply-supported and clamped composite laminates by using the 3-D theory of elasticity and the differential quadrature discretization method. Messina [12] investigated the influence of boundary conditions on the free vibrations of cross-ply laminated rectangular plates by using a Ritz-type formulation. Narita [13] analyzed the free vibration behavior of a single-layer rectangular parallelepiped made of unidirectionally fiber-reinforced composites using 3-D Ritz method, in which the displacement components of the parallelepiped were expanded as a product of power series and basic functions. Li et al. [14] analyzed the 3-D free vibration problems of functionally graded material sandwich rectangular plates with simply supported and clamped edges by using the Ritz method, in which the displacements of rectangular plates were expanded by Chebyshev polynomial series multiplied by boundary functions. Based on a variational method, Heyliger [15] investigated the free vibration behaviors of layered elastic and piezoelectric rectangular parallelepipeds with traction-free boundary conditions. Civalek and Baltacioğlu [16] introduced the discrete singular convolution method to the free vibration analysis of composite thick plates.

The prediction of displacement and stress fields for multilayered structures under external dynamic loads plays an important role in the stage of structure design. However, very few theoretical studies have been reported on forced vibration analyses of multilayered parallelepipeds. Kapuria and Achary [17] presented an exact 3-D solution for steady-state vibration analysis of simply-supported piezoelectric rectangular plates subjected to harmonic electromechanical loads. Later, Kapuria and Nair [18] analyzed the steady-state harmonic responses of simply supported cross-ply piezoelectric laminated rectangular plates with interlaminar bonding imperfections. Using the Laplace transform method and the elasticity theory, Kardomateas et al. [19] investigated the transient vibration responses of sandwich beams and plates consisting of orthotropic core and face sheets subjected to blast loading. More recently, the first author and coworkers [20] presented a modified variational method for predicting the 3-D free and transient vibrations of composite rectangular parallelepipeds with arbitrary combinations of boundary conditions. This method offers a simple yet powerful alternative to other analytical and numerical techniques for providing accurate vibration solutions of composite beams, plates and solids. However, their analyses were restricted to single-layered rectangular parallelepipeds.

The forgoing brief literature survey reveals that a general 3-D elasticity solution for the free vibration analyses of composite laminated and sandwich rectangular parallelepipeds (including beams, plates and solids) with boundary conditions of arbitrary type does not seem to exist. Moreover, analytical or numerical analyses for transient vibrations of multilayered parallelepipeds excited by arbitrary time-dependent forces have not been attempted so far. Considering the above aspects in view, free and transient vibrations of multilayered rectangular parallelepipeds are analyzed in this paper based on the 3-D elasticity theory and using an efficient variational method recently developed by the first author and coworkers [20]. Multilevel partitioning hierarchy, viz., multilayered parallelepiped, individual layer and layer segment, is employed in the analysis. The appropriate continuity constraints on common interfaces are imposed by means of a modified variational principle. The displacement field of each layer segment is approximated as the product of orthogonal polynomials and/or trigonometric functions. For a rectangular parallelepiped simply supported at one pair of opposite edges, the present method can be implemented in a semi-analytical sense. This is achieved by expressing the displacement field of each layer segment in terms of trigonometric functions in the coordinate direction between the simply supported edges, and by employing orthogonal polynomials with respect to the coordinate direction between the other two edges. Numerical examples are carried out for composite laminated and sandwich beams, plates and solids with various geometrical and material parameters, and extensive comparison is made with other methods. New vibration results are also presented, which may serve as benchmark solutions for future investigations. Regarding the transient vibration problems, the time-histories of displacement and stress responses for multilayered rectangular parallelepipeds subject to several impulsive loads, including a rectangular pulse, a triangular pulse, a half-sine pulse and an exponential pulse, are also investigated.

2. Theoretical formulations

2.1. Model description

Consider a multilayered composite rectangular parallelepiped composed of N_l perfectly bonded layers, with each layer having independent thickness and material properties. A typical sandwich construction consisting of three principal layers, i.e., two thin, stiff facing layers and a thick, flexible core, is treated as a special case of the general multilayer configuration. The geometrical dimensions (length L, width B and thickness H) and the coordinate system of a multilayered rectangular parallelepiped are illustrated in Fig. 1. The kth layer (start the counting from the bottom) has a thickness of $h_k = z_{k+1} - z_k$, where z_{k+1} and z_k are the thickness coordinates of the upper and lower surfaces of the layer. The principle material axes of each layer in the rectangular parallelepiped are denoted as the 1, 2 and 3 axes, and for the kth layer, the 1-direction may lie at an angle $\theta^{(k)}$ with respect to the x coordinate. It is assumed that the two opposite faces $y = 0, B$ of the parallelepiped are either free of displacement constraints or simply-supported, whereas any combination of boundary conditions may be prescribed for the other two faces at $x = 0, L$.

In the Cartesian coordinate system, the linear strain–displacement relations for the kth layer are given as

$$\varepsilon_x^{(k)} = \frac{\partial u^{(k)}}{\partial x}, \quad \varepsilon_y^{(k)} = \frac{\partial v^{(k)}}{\partial y}, \quad \varepsilon_z^{(k)} = \frac{\partial w^{(k)}}{\partial z}, \quad \varepsilon_{yz}^{(k)} = \frac{\partial v^{(k)}}{\partial z} + \frac{\partial w^{(k)}}{\partial y},$$
$$\varepsilon_{xz}^{(k)} = \frac{\partial w^{(k)}}{\partial x} + \frac{\partial u^{(k)}}{\partial z}, \quad \varepsilon_{xy}^{(k)} = \frac{\partial u^{(k)}}{\partial y} + \frac{\partial v^{(k)}}{\partial x} \tag{1.a-f}$$

where $u^{(k)}, v^{(k)}$, and $w^{(k)}$ respectively denote the displacement components of an arbitrary point in the x, y and z directions. $\varepsilon_x^{(k)}, \varepsilon_y^{(k)}$, $\varepsilon_z^{(k)}, \varepsilon_{yz}^{(k)}$, $\varepsilon_{xz}^{(k)}$ and $\varepsilon_{xy}^{(k)}$ represent the normal and shear strain components.

The linear constitutive equations valid for the nature of the kth orthotropic layer are given by