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Nonlinear electro-mechanical responses of functionally graded piezoelectric beams

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ABSTRACT

This study presents analyses of the nonlinear electro-mechanical responses of functionally graded piezoelectric beams undergoing small deformation gradients. The studied functionally graded beams comprise of electro-active and inactive constituents with gradual compositions varying through the thickness of the beams. Two types nonlinear electro-mechanical responses are considered for the active constituents, which are nonlinear electro-mechanical behaviors for the polarized piezoelectric constituent under electric fields smaller than the coercive limit, and polarization switching responses due to cyclic electric fields with high amplitude. The inactive constituent is modeled with uncoupled linear electro-elastic response. The functionally graded beam is discretized into several graded layers through its thickness. Each layer is comprised of different compositions of the active (piezoelectric) inclusions and conductive matrix. A particle-unit-cell micromechanical model is used to obtain the nonlinear electro-mechanical responses in each layer and is integrated within the laminate theory in order to obtain the overall nonlinear electro-mechanical responses of the functionally graded piezoelectric beams. The numerical predictions are compared with experimental data available in literature. Parametric studies are then performed in order to examine the effects of the thickness of the beam, of the concentration of the constituent, and the frequency of the cyclic electric field on the overall electro-mechanical response of the functionally graded piezoelectric beams.

1. Introduction

A piezoelectric bimorph beam, which is formed by bonding two thin layers of piezoelectric ceramic plates, is capable of attaining bending (lateral) displacements which found applications in actuators. An actuating mechanism is induced by applying electric fields through the thickness of the piezoceramics plate so that one of the piezoelectric ceramic plates would elongate in the longitudinal direction and the other plate would contract in the longitudinal direction. The relatively large differences in the deformations, i.e., tension and compression, in the longitudinal direction of the two piezoelectric ceramic plates lead to high stress discontinuities at the interfaces between the piezoelectric plates. The high stress discontinuities at the interface can cause debonding and eventually failure of the piezoelectric bimorph beam. Functionally graded piezoelectric beams (FGPBs) have been considered as one of the promising solutions to relieve the high stress discontinuities at the interface of the piezoelectric ceramic plates by introducing gradual changes in the compositions of the piezoelectric ceramic plates, see for example, Wu et al. [34], Jin and Meng [16], Takagi et al. [30], Li et al. [19], Alexander and Brei [4], and Hudnut et al. [15].

There have been studies on analyzing the electro-mechanical responses of functionally graded piezoelectric structures. Most studies have been focused on the linear electro-mechanical responses, which are applicable when the piezoelectric structures are subjected to a relatively small electric field inputs. Examples of the analytical and numerical (finite element) studies of the response of piezoelectric functionally graded structures can be found in Hauke et al. [13], Wang and Noda [33], Bhangale and Ganesa [6], Carbonari et al. [7], Carbonari et al. [8], Malakooti and Sodano [24], etc. Carbonari et al. [7] used a topology optimization method in order to determine the optimal gradations and polarization axes in the functionally graded bimorph beam, which is measured by maximizing the output lateral displacements. They considered a combination of piezoelectric ceramics (PZT5A) and metals (gold) as the constituents in the functionally graded bimorph beam. Vatanable et al. [32] also used a topology optimization to investigate the influence of a pattern gradation on the design of FGPBs for energy harvesting devices. Several studies have considered smooth and monotonic spatial functions in order to determine the overall electro-mechanical properties in the graded

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piezoelectric materials, e.g., Bhangale and Ganesa [6], Huang et al. [14], Panda and Ray [28], Yang [35]. Using the monotonic function in determining the overall material properties along the graded location is often convenient when linear electro-mechanical responses are considered. In several composites, some of the effective electro-mechanical properties can be well approximated by using rule of mixture of the properties of the constituents.

In actuation applications, large electric fields are often applied to the piezoelectric composites, leading to nonlinear electro-mechanical responses. Several experimental studies have shown nonlinear electro-mechanical and hysteretic responses in piezoelectric functionally graded beams, i.e., Kouvatov et al. [17], Jin and Meng [16]. The studied showed nonlinear polarization and switching responses when the beams are subjected to relatively high electric fields. Takagi et al. [30] also showed nonlinear electro-mechanical responses in a polarized functionally graded beam under large electric fields. When a nonlinear behavior is exhibited by one or more of the constituents in the composites, a simple rule of mixture may not be capable in predicting the overall nonlinear responses of the composites. Several micromechanics models that include various microstructural arrangements and different constituent behaviors have been widely used in predicting both linear and nonlinear electro-mechanical responses of piezoelectric composites, e.g., Newnham et al. [27], Dunn and Taya [11], Chen [9], Aboudi [1], Aboudi [2], Muliana [25], Lin and Muliana [20], Lin and Muliana [21]. To the best knowledge of the authors, analyzing nonlinear electro-mechanical responses, including polarization and hysteretic behaviors, of functionally graded piezoelectric structures is currently limited. Kouvatov et al. [17] have analyzed nonlinear hysteretic responses of functionally graded beam. The beam is discretized into several layers of homogeneous materials and a higher order polynomial function is considered in order to capture the nonlinear hysteretic polarization in each electro-active layer of the functionally graded beam. However, it is not quite clear on how they incorporated the hysteretic responses using the higher order polynomial function.

This study presents analyses of the nonlinear electro-mechanical response of polarized FGPBs and polarization switching behavior in FGPBs, undergoing small deformation gradients. The lateral deformations in the FGPBs are due to applications of electric fields through the thickness of the beams. Since the FGPBs are assumed slender and to undergo small deformation gradient, the Euler–Bernoulli beam theory is used to describe the deformations of FGPBs. The functionally graded beam is discretized into several graded layers through its thickness as shown in Fig. 1. The interfaces between layers are assumed perfectly bonded so that the charge continuously flows through the thickness, i.e., charges in every layer of the FGPBs are assumed uniform. Each layer is comprised of different compositions of the active (piezoelectric) inclusions and conductive matrix, i.e., metallic matrix. The microstructure in each layer is idealized as solid spherical inclusions randomly distributed in a homogeneous matrix and it is assumed that all particles are fully surrounded by homogeneous matrix without any contents of pores and defects. A unit-cell model consisting of eight inclusion and matrix subcells is then considered for obtaining the overall (homogenized) electro-mechanical properties and responses in each layer. The inclusions and the matrix are assumed perfectly bonded and the spatial variations of the field variables in each subcell are assumed uniform. The piezoelectric constituents are modeled as nonlinear electro-mechanical responses while the matrix is assumed linear elastic. The nonlinear responses are due to an application of large electric fields. Two nonlinear electro-mechanical constitutive models are considered for the piezoelectric constituent. The first model is a nonlinear electro-mechanical relation proposed by Tiersten [31] for a polarized piezoelectric material under electric fields smaller than the coercive limit, and the second model takes into account the hysteretic polarization switching responses due to large electric field inputs, previously studied by Muliana [26] and Sohrabi and Muliana [29]. The manuscript is organized as follows. Section 2 briefly presents the constitutive models used for the piezoelectric constituent and the unit-cell model for obtaining the homogenized nonlinear electro-mechanical responses. Section 3 discusses the analyses of functionally graded piezoelectric beam. Numerical results and parametric studies are given in Section 4. Finally Section 5 is dedicated to concluding remarks.

2. Constitutive models for the constituents and unit-cell model

2.1. Electro-mechanical constitutive models

Piezoceramics are polarized by applying high electric field, above the coercive electric field at elevated temperatures [23] before they are used as sensors and actuators. When high electric field is prescribed to the polarized piezoceramics, which is often the case in actuator applications, they exhibit nonlinear electromechanical coupling response. Tiersten [31] presented a constitutive model for analyzing nonlinear electro-mechanical responses of polarized piezoceramics subject to a large electric field but smaller than coercive electric field of the piezoceramics. Another type of nonlinear electromechanical coupling response is the hysteretic polarization switching response, which can occur when the ferroelectric materials are subjected to cyclic electric field with amplitude above the coercive electric field.

A nonlinear constitutive model proposed by Tiersten [31] for polarized piezoceramics undergoing large electric fields and small strains is:

$$\varepsilon_{ij} = s_{ijkl}\sigma_{kl} + d_{kij}E_k + \frac{1}{2}f_{klij}E_lE_k, \tag{2.1}$$

$$D_i = d_{ikl}\sigma_{kl} + \kappa_{ij}E_j + \frac{1}{2}\chi_{ijk}E_kE_j, \tag{2.2}$$

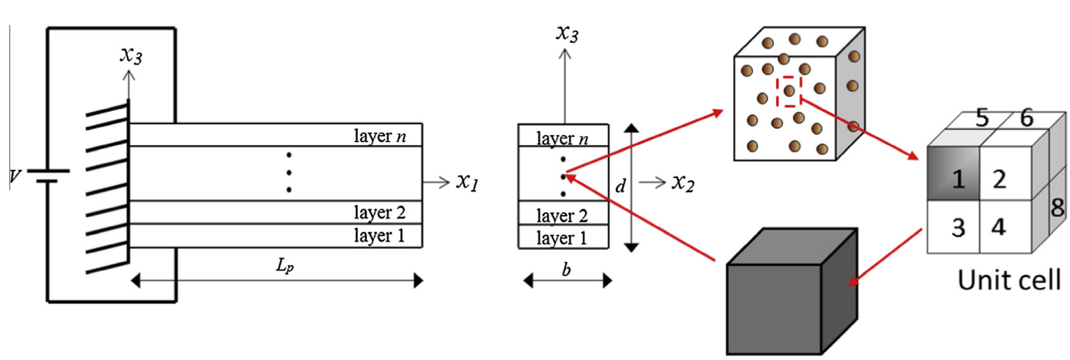

Fig. 1. Schematic of a functionally graded piezoelectric beam and its microstructural approximation.