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High-order free vibration analysis of sandwich plates with both functionally graded face sheets and functionally graded flexible core

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ABSTRACT

Free vibration analysis of functionally graded material sandwich plates is studied using a refined higher order sandwich panel theory. A new type of FGM sandwich plates, namely, both functionally graded face sheets and functionally graded flexible core are considered. The functionally graded material properties follow a power-law function. The first order shear deformation theory is used for the face sheets and a 3D-elasticity solution of weak core is employed for the core. On the basis of continuities of the displacements and transverse stresses at the interfaces of the face sheets and the core, equations of motion are obtained by using Hamilton's principle. The accuracy of the present approach is validated by comparing the analytical results obtained for a degradation model (functionally graded face sheets and homogeneous flexible core) with ones published in the literatures, as well as the numerical results obtained by finite element method and good agreements are reached. Then, parametric study is conducted to investigate the effect of distribution of functionally graded material properties, thickness to side ratio on the vibration frequencies.

1. Introduction

Functionally graded materials (FGMs) are novel multi-functional materials which were first introduced by Japanese scientists in 1984 [1,2]. FGMs vary continuously and gradually over volume in composition and microstructure for the specific purpose of controlling variations in thermal, structural or functional properties. In spite of microscopically inhomogeneous FGMs are macroscopically homogeneous composites usually made from a mixture of metals and ceramics. These advantages help to reduce mechanically and thermally induced stresses due to the material property mismatch and to improve the bonding strength. Therefore, there are a broad range of applications such as biomechanical, automotive, aerospace, mechanical, civil, and naval engineering and wide researches on FGMs [3].

Approaches such as using shear deformation plate theory, energy method, finite-element method, and three-dimensional elastic theory were carried out to study mechanical behavior of the FGM structures while the material properties of FGM were usually assumed to vary exponentially through the thickness. Reddy [4] presented the solutions of static behavior for the FGM rectangular plates based on his third-order shear deformation plate theory. Cheng and Batra [5] presented the results for the buckling and steady state vibrations of a simply supported FGM polygonal plate based on Reddy's plate theory. Loy et al. [6] presented Rayleigh–Ritz solutions for free vibration of simply supported cylindrical shells made of an FGM compound of stainless steel and nickel by using Love's shell theory. Praveen and Reddy [7] investigated the nonlinear static and dynamic response of functionally graded ceramic–metal plates using finite element method that accounts for the transverse shear strains, rotary inertia and moderately large rotations in the von Karman sense. Vel and Batra [8] presented a three-dimensional exact solution for free and forced vibrations of simply supported FGM rectangular plates by using suitable displacement functions to reduce equations governing steady state vibration of the plate. Ye et al. [9] studied free vibration of laminated functionally graded spherical shells with general boundary conditions and arbitrary geometric parameters by using three-dimensional shell theory. Beena and Parvathy [10] explored the extension of the spline finite strip method to the linear static analysis of functionally graded material plates. Mantari et al. [11] presented a free vibration analysis of functionally graded plates (FGPs) resting on elastic foundation. The displacement field was based on a novel non-polynomial higher order shear deformation theory (HSDT). Pradhan and Chakraverty [12] studied the free vibration analysis of functionally graded material (FGM) beams subjected to different

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sets of boundary conditions. Tornabene et al. [13] investigated the free vibrations of free-form doubly-curved shells made of functionally graded materials using higher-order equivalent single layer theories.

In recent years FGMs are used in sandwich structures to alleviate the large interfacial shear stress concentration due to the stiffness mismatching at the face sheets–core interfaces. The gradual variation of material properties of FGMs ensures the continuity of material properties at the face sheets–core interfaces. The construction commonly exists in two types: FGM face sheet–homogeneous core and homogeneous face sheet–FGM core. Zenkour [14,15] presented a two-dimensional solution for bending, buckling, and free vibration analysis of simply supported functionally graded ceramic–metal sandwich plates based on the sinusoidal shear deformation plate theory. In his study, the sandwich plate faces are made of functionally graded material, the core layer is made of the isotropic ceramic material. Woodward and Kashtalyan [16] presented the 3D elasticity solution for bending response of sandwich plates with functionally graded core. Liu et al. [17] investigated the dynamic responses and blast resistance of all-metallic sandwich plates with functionally graded close-celled aluminum foam cores using finite element simulations and compared with those of ungraded single-layer sandwich plates. Shodja et al. [18] developed an exact thermoelasticity solution for sandwich structures with functionally graded coating. A fourth order inhomogeneous partial differential equation was obtained and solved exactly using Fourier series method. Shen and Li [19] presented compressive postbuckling under thermal environments and thermal postbuckling due to heat conduction for a simply supported sandwich plate with FGM face sheets. The governing equations were developed based on a higher order shear deformation plate theory that includes thermal effects. Dozio [20] dealt with the formulation of advanced two-dimensional Ritz-based models for accurate prediction of natural frequencies of thin and thick sandwich plates with core made of functionally graded material. Sobhy [21] studied the vibration and buckling behavior of exponentially graded material (EGM) sandwich plate resting on elastic foundations under various boundary conditions. The EGM sandwich plate was assumed to be made of a fully ceramic core layer sandwiched by metal/ceramic EGM coat. The governing equations were deduced by using various shear deformation plate theories. Using first order shear deformation theory, Sofiyev [22] discussed the vibration and buckling of sandwich cylindrical shells covered by different types of coatings, such as functionally graded, metal and ceramic coatings and subjected to a uniform hydrostatic pressure. Sofiyev and Kuruoglu [23] investigated the vibration and buckling of functionally graded orthotropic cylindrical shells under external pressures using the shear deformation shell theory (SDST). The basic equations of shear deformable FG orthotropic cylindrical shells are derived by using Donnell shell theory and solved by using the Galerkin method. Xiang et al. [24] analyzed the free vibration of sandwich plate with functionally graded face and homogeneous core by using meshless global collocation method based on the thin plate spline radial basis function and nth-order shear deformation theory.

In general, the core layer of sandwich structure is usually flexible so that it is necessary to take transverse compression into account. Then it is a more complicated problem to deal with the vibration of FGM sandwich structure with flexible core. Some methods such as Ritz method, high-order sandwich panel theory were used by several researchers. Li et al. [25] studied the free vibration of functionally graded material sandwich rectangular plates with simply supported and clamped edges based on Ritz method. Both two FGM sandwich construction types were studied and the flexibility of the core was considered. Using the high-order sandwich panel theory, Rahmani et al. [26] analyzed free vibration of sandwich beam with a flexible functionally graded syntactic core. Khalili and Mohammadi [27] studied the free vibration of sandwich plates with temperature-dependent functionally graded face sheets in various thermal environments by using an improved high-order sandwich plate theory, which considered both the in-plane stresses and transverse stresses of the core in the formulation. Neves et al. [28,29] analyzed the free vibration of functionally graded shells by using a higher-order shear deformation theory and radial basis functions collocation, accounting for through-the-thickness deformations.

However, the study is rather limited about a novel type of FGM sandwich structure at present, namely, both functionally graded face sheets and functionally graded flexible core. In the present study, this type of FGM sandwich plates is considered. The free vibration of the FGM sandwich plate is analyzed by using a refined higher order sandwich panel theory, in which the first order shear deformation theory is used for the face sheets and a 3D-elasticity solution of weak core [30,31] is employed for the core. On the basis of continuity of the displacements and transverse stresses at the interfaces of the face sheets and the core, equations of motion are derived by Hamilton's principle. By the higher order sandwich panel theory, the localized bending effects associated with local changes of the FGM sandwich panel thickness can be obtained.

2. Theoretical formulation

2.1. Geometrical configuration and basic assumption

The FGM sandwich plate in this study is composed of two FGM face sheets and a flexible FGM core, as shown in Fig. 1. The thicknesses of the three layers, "Layer 1", "Layer 2", and "Layer 3", namely, the bottom face sheet, the core layer, and the top face sheet, are $2d_b$, $2c$, and $2d_t$ respectively. Indices t and b refer to the top and the bottom face sheets of the plate, respectively. For simplicity, the ratio of the thickness of each layer from bottom to top is denoted by the combination of three numbers, i.e. "1–2–1", "1–1–1" and so on. The first order shear deformation theory is applied to the face sheets with transverse shear stresses

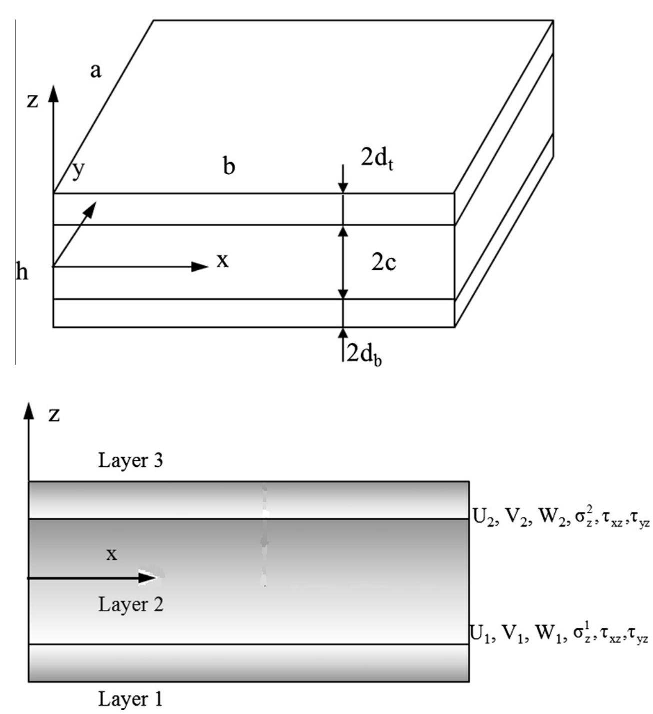

Fig. 1. Geometry of the FGM sandwich plate and the material variation along the thickness.