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On a problem of the vibration of functionally graded conical shells with mixed boundary conditions

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ABSTRACT

This paper presents a theoretical approach to solve vibration problems of functionally graded (FG) truncated conical shells under mixed boundary conditions. The material properties of FG shell are assumed to vary continuously through the thickness of the conical shell. The fundamental relations, motion and strain compatibility equations of FG truncated conical shells are derived by means of the Airy stress function method. Two cases of mixed boundary conditions are investigated. The basic equations are solved by using Galerkin method and fundamental cyclic frequencies of FG truncated conical shells are obtained. The results are compared and validated with the results available in the literature. The detailed parametric studies are carried out to investigate the influences of radius-to-thickness ratio, lengths-to-radius ratio, material composition and mixed boundary conditions on the fundamental cyclic frequencies of truncated conical shells.

1. Introduction

Functionally graded materials (FGMs) with continuously varying material properties have attracted much attention of researchers because of their excellent performance [1,2]. FGMs are used in missiles engine, resistant coatings in space plans, atomic reactors, spacecraft thermal shields, intelligent electrical components, submarines, turbine components, sensors and others [3]. FGMs are fabricated by combining disparate materials that are graded in the thickness direction with variations in constituent volume fractions using various methods, such as powder metallurgy methods vapor deposition techniques including chemical and physical vapor deposition, centrifugal methods including centrifugal casting and centrifugal solid, and solid free-form methods [4]. Therefore, FGMs possess noticeable advantages over homogeneous and layered materials in maintaining the integrity of the structure. So far, there have been a very large number of researches available both in theoretical analysis, numerical simulations, and experimental observations. A detailed review on the performance of FGM can be seen in the literature [5–7].

The circular conical shells have been widely used in a variety of engineering fields as important structural components due to their special geometric shapes, especially in aircrafts, ships, rockets, submarines, missile bodies, pressure vessels, oil tanks and buildings. Comprehensive works on the dynamic responses of conical shells have been reported in the literature. An earlier survey on the free vibration of conical shells was provided by Leissa [8] and Volmir [9], the effects of different boundary conditions and semi-vertex angles on the frequency characteristics of conical shells were investigated. It is difficult to obtain the closed-form solutions for truncated conical shells with various boundary conditions. Therefore, numerical or approximate methods, such as finite-element method, finite-difference method, weighted residual method and Rayleigh–Ritz method, have been widely used to obtain the approximate solutions [10–22]. The above mentioned studies examined vibration of cylindrical and conical shells with different boundary conditions, such as simply supported, free and clamped boundary conditions, or combinations thereof.

Due to the increased relevance of FGMs in the design of engineering structures, many studies have been reported on the vibration analysis of FGM shells with different boundary conditions. The first work on the vibration analysis of an FGM cylindrical shell was reported by Loy et al. [23], which analyzed the natural frequency characteristics by constituent different volume fractions under simply supported boundary condition. Pradhan et al. [24] studied effects of boundary conditions and volume fractions (power law exponent) on the natural frequencies of the FG cylindrical shell made up of stainless steel and zirconium. Patel et al. [25] used the finite element method for free vibration analysis of

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FGM elliptical cylindrical shells. Zhi and Hua [26] evaluated natural frequencies of an FGM cylindrical shell with cavities and effects of boundary conditions. Iqbal et al. [27] used wave propagation for FGM cylindrical shells with the influence of boundary conditions. Isvandzibaei et al. [28] studied effects of uniform interior pressure distribution on vibration of FGM cylindrical shell with rings support subjected to ten boundary conditions. Some recent research presented on the vibration of FGM conical shells under different boundary conditions can be found in the literature. For example, Sofiyev [29] presented the vibration and stability behavior of freely supported FGM conical shells subjected to an external pressure. Tornabene [30] examined free vibration analysis of functionally graded conical, cylindrical shell and annular plate structures with a four-parameter power-law distribution with fully clamped or free edge boundary conditions. Sofiyev and Kuruoglu [31] studied vibration analysis of FGM truncated and complete conical shells resting on elastic foundations under clamped or freely supported boundary conditions. Malekzadeh et al. [32] investigated three dimensional free vibration of FG truncated conical shells subjected to thermal environment under different boundary conditions. Liew et al. [33] investigated postbuckling responses of functionally graded cylindrical shells under axial compression and thermal loads. Dung et al. [34] investigated instability of eccentrically stiffened functionally graded truncated conical shells under mechanical loads. Malekzadeh and Heydarpour [35] investigated the influences of centrifugal and Coriolis forces in combination with the other geometrical and material parameters on the free vibration behavior of rotating functionally graded (FG) truncated conical shells subjected to different boundary conditions. Abediokhchi et al. [36] presented bending analysis of moderately thick FG conical panels with various boundary conditions using GDQ method. Pradhan and Chakraverty [37] examined free vibration of Euler and Timoshenko functionally graded beams by Rayleigh–Ritz method. Sahmani et al. [38] investigated dynamic stability analysis of functionally graded higher-order shear deformable micro shells based on the modified couple stress elasticity theory. Akbari et al. [39] presented free vibration of FGM Lévy conical panels with various boundary conditions. Heydarpour et al. [40] investigated free vibration of FG truncated conical shells under internal pressure with various boundary conditions. Su et al. [41] studied three-dimensional vibration analysis of thick FG conical, cylindrical shell and annular plate structures with arbitrary elastic restraints. Idesman [42] presented accurate finite-element modeling of wave propagation in composite and functionally graded materials. Huang and Han [43] investigated elastoplastic buckling of axially loaded functionally graded material cylindrical shells. Tornabene et al. [44,45] studied vibration, stress and strain recovery for functionally graded free-form and doubly-curved sandwich shells using higher-order equivalent single layer theory. Apuzzo et al. [46] presented some analytical solutions of functionally graded Kirchhoff plates.

From the literature survey, one can see that there are many studies on the vibration of FGM conical shells with mixed boundary conditions, such as clamped-free, clamped-simply supported, and simply supported-free. In the present study, the vibration of FGM circular conical shells examined under the following mixed boundary conditions. At one end of a FG truncated conical shell holds free support, and the other end is in a sleeve that prevents its longitudinal displacement and rotation. The governing equations of motion are expressed as functions of two kinematic parameters, by using the constitutive and kinematic relationships. Then using Galerkin's method, these equations have been transformed to a pair of time dependent differential equation and fundamental cyclic frequency of free vibration for FG truncated conical shells with two mixed boundary conditions is obtained. Results for various states are verified with the known data in the literature. Finally, the detailed parametric studies are carried out to study the influences of variations of the semi-vertex angle, radius-to-thickness ratio, lengths-to-radius ratio and material composition on the cyclic frequency of conical shells.

2. Formulation of the problem

The schematic configuration of an FGM truncated conical shell and coordinate system ($S\theta\zeta$) are shown in Fig. 1, where the S-axis in the direction of the generator of the cone, the ζ-axis in the direction normal to the reference surface of the cone, and θ-axis in the direction "perpendicular" to the $S-\zeta$ plane. R_1 and R_2 indicate the radii of the cone at its small and large ends, respectively, γ denotes the semi-vertex angle of the cone, L is the length and h is the thickness of the truncated conical shell. The distances along the generator from the apex to the small and large ends of the truncated conical shell are S_1 and $S_2 = S_1 + L$, respectively.

It is assumed that the FGM is made of a mixture of metal and ceramic phases, and the material composition varying smoothly along the axis-ζ, only. The variation of the Young's modulus, Poisson's ratio and density of the FGM shell are

$$E(\zeta_1) = (E_c - E_m)V(\zeta_1) + E_m,\ \nu(\zeta_1) = (\nu_c - \nu_m)V(\zeta_1) + \nu_m,$$
$$\rho(\zeta_1) = (\rho_c - \rho_m)V(\zeta_1) + \rho_m \tag{1}$$

where E_m, ν_m, ρ_m and E_c, ν_c, ρ_c are the Young's modulus, Poisson's ratio and density of the metal and ceramic surfaces of the FGM shell, respectively. The index "m" represents the metal phase, while the index "c" represents the ceramic phase.

The compositional gradation of the FGM conical shell is defined by the volume fraction of the ceramic phase and the following functions of $V(\zeta_1)$ will be considered [29]:

1. Linear : $V(\zeta_1) = \zeta_1 + 0.5, \quad \zeta_1 = \zeta/h$ (2.1)
2. Quadratic : $V(\zeta_1) = (\zeta_1 + 0.5)^2$ (2.2)
3. Inverse Quadratic : $V(\zeta_1) = 1 - (0.5 - \zeta_1)^2$ (2.3)
4. Cubic : $V(\zeta_1) = 3(\zeta_1 + 0.5)^2 - 2(\zeta_1 + 0.5)^3$ (2.4)

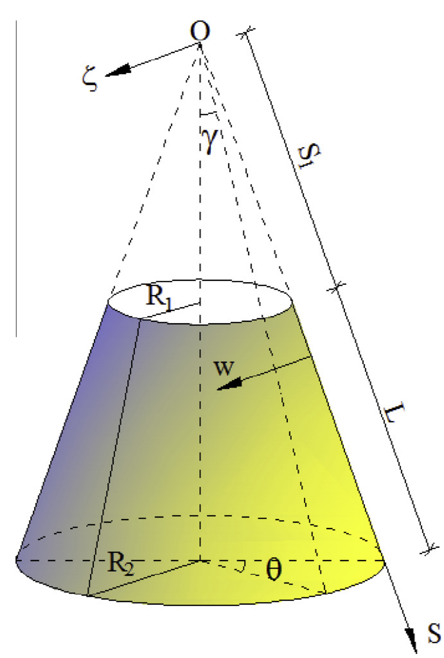

Fig. 1. The schematic configuration of an FGM conical shell and coordinate system.