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Transverse wakefields due to asymmetric protrusions into a vacuum chamber

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ABSTRACT

We analyze the effect of a wakefield caused by an asymmetric protrusion inside the accelerator vacuum chamber. The asymmetry leads to a transverse kick on the beam and an increase of the projected transverse beam emittance. Calculations are done for a model rectangular protrusion in a vacuum chamber of rectangular cross-section. Based on our analysis, numerical estimates are given for the SuperKEKB accelerator in KEK, Japan, and TLEP-W proposal at CERN.

1. Introduction

Traditionally wakefield calculations are focussed on elements of the vacuum chamber which have a certain degree of symmetry such as an axisymmetry and top-down or right-left symmetry. Such elements have a property that the transverse wakefield vanishes if the beam propagates along the symmetry axis of the system.[1] The wakefields appear only when the beam is offset from the axis, and result in a deflection of the beam which is proportional to the offset. They are the source of transverse instabilities in the beam.

In this paper we attract attention to another type of wakefield that is caused by asymmetric protrusions inside of a vacuum chamber. The protrusions can be parts of elements of the vacuum chamber such as masks, bellows and beam position monitors. The asymmetry leads to a transverse kick *even if the beam is not offset from the axis of the machine*. It does not lead to a beam instability, but results in an increase in projected transverse beam emittance. Given that in modern electron and positron rings the vertical emittance is extremely small, such protrusions can set a limit on the minimally achievable transverse emittance in a given accelerator and lead to a decrease in luminosity of a collider, or a deterioration of the transverse coherence properties in a ring light source.

An effect similar to the one discussed in this paper was earlier studied in Ref. [1], where it was assumed that the transverse kick was due to an offset of the beam orbit from the nominal position. In principle such a distortion can be cured by the proper orbit correction, while the effect described in this paper is not sensitive to a shift in orbit.

The strength of the transverse kick depends on the geometry of the protrusion and the parameters of the beam. In this paper, which is aimed at demonstrating the basic effect, we choose the simple model of a rectangular protrusion in a rectangular vacuum chamber. Even for such a relatively simple geometry the calculation of the wakefield, in general, is too difficult for an analytical treatment, and requires a 3D computer simulation. Only in the limit when the bunch length is sufficiently small can one use the optical model developed in Refs. [2,3] to calculate the wakefield analytically.

This paper is organized as follows. In Section 2 we specify the geometry of the protrusion and discuss the effect of its wakefield on the effective beam emittance. In Section 3 we present the results of the calculations using the optical model, with the details of the calculations given in Appendix A. In Section 4 we present the results of computer simulations with GdfidL [4] and estimate the effect for parameters of SuperKEKB. In Section 5 we give an estimate for TLEP-W accelerator. We conclude this paper by Section 6 with a summary of our results.

2. Effect of transverse wake on beam emittance

We first consider a general case where there are localized impedance sources at several locations in the ring. Source i is located at position s_i and generates a transverse (vertical) wake

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[1] We are not considering here the so-called quadrupole wake in non-axisymmetric systems that leads to the differential focusing of the beam even when it propagates on the axis.

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$W_y^{(i)}(z)$, where z is the longitudinal coordinate in the bunch with positive z corresponding to the head of the bunch. We can calculate the vertical deflection angle of the beam due to source i as

$$\theta^{(i)}(z) = \frac{eQW_y^{(i)}(z)}{\gamma mc^2} \tag{1}$$

where Q is the bunch charge and γmc^2 is the beam energy. In a ring with a given vertical function $\beta(s)$, where s is the coordinate along the ring circumference, a localized kick (1) would lead to an orbit distortion, in which each slice of the bunch executes a different betatron oscillation. This trajectory of a slice at coordinate z is given by the standard formula for orbit distortion in a ring (see, e.g., [5])

$$\Delta y(s,z) = \frac{\sqrt{\beta(s)}}{2\,\sin(\pi\nu)}\sum_i \theta^{(i)}(z)\sqrt{\beta(s_i)}\,\cos(\psi(s)-\psi(s_i)-\pi\nu) \tag{2}$$

where $\psi(s)$ is the betatron phase, ν is the vertical tune, and the summation goes over all impedance sources. Correspondingly, there is a z-dependent slope $\Delta y'(s,z)$ given by

$$\Delta y'(s,z) = \frac{\beta'(s)}{4\sqrt{\beta(s)}\,\sin(\pi\nu)}\sum_i \theta^{(i)}(z)\sqrt{\beta(s_i)}\,\cos(\psi(s)-\psi(s_i)-\pi\nu) - \frac{1}{2\sqrt{\beta(s)}\,\sin(\pi\nu)}\sum_i \theta^{(i)}(z)\sqrt{\beta(s_i)}\,\sin(\psi(s)-\psi(s_i)-\pi\nu). \tag{3}$$

Due to this distortion of the beam shape, its emittance increases by some amount $\Delta\epsilon$. Assuming that the increase is small, $\Delta\epsilon$ is given by the following formula:

$$\Delta\epsilon = \frac{1}{2}\left[\frac{1+\alpha^2}{\beta}\langle(\Delta y-\langle\Delta y\rangle)^2\rangle + \beta\langle(\Delta y'-\langle\Delta y'\rangle)^2\rangle + 2\alpha\langle(\Delta y-\langle\Delta y\rangle)(\Delta y'-\langle\Delta y'\rangle)\rangle\right] \tag{4}$$

where the angular brackets indicate averaging with the longitudinal distribution function of the beam and $\alpha(s) = -\beta'(s)/2$.

Eqs. (2)–(4) can be used for practical calculations but are too complicated for general analysis of the magnitude of the effect. In what follows, we consider a simple case of a single localized source of impedance $W_y(z)$ located at coordinate s_0 where $d\beta/ds|_{s=s_0} = 0$. Eq. (4) then simplifies to

$$\Delta\epsilon = \frac{1}{2}\left[\frac{1}{\beta_0}\langle(\Delta y_b-\langle\Delta y_b\rangle)^2\rangle + \beta_0\langle(\Delta y'_b-\langle\Delta y'_b\rangle)^2\rangle\right] \tag{5}$$

where $\beta_0 = \beta(s_0)$ and $\Delta y_b(z) = \Delta y(s_0, z)$ and $\Delta y'_b(z) = d\Delta y(s,z)/ds|_{s=s_0}$. Using $d\beta/ds|_{s=s_0} = 0$ from (2) and (3) we obtain

$$\Delta y_b(z) = \frac{\theta(z)}{2}\beta(s_0)\cot(\pi\nu), \quad \Delta y'_b(z) = \frac{\theta(z)}{2}. \tag{6}$$

Substituting (6) into (5) we obtain

$$\Delta\epsilon = \tfrac{1}{8}\beta_0\theta_{\rm rms}^2\csc(\pi\nu)^2 \tag{7}$$

where the rms angular spread of θ is given by

$$\theta_{\rm rms} = \frac{e}{\gamma mc^2}\langle(W_y-\langle W_y\rangle)^2\rangle^{1/2}. \tag{8}$$

In the next section we will show how the impedance of a rectangular protrusion can be calculated using the optical model and apply this impedance to evaluating the projected beam emittance increase.

3. Geometry of protrusion and transverse wakefields in optical model

We consider a vacuum chamber with rectangular cross-section $a\times b$ and the reference orbit of the beam located at the center of the chamber, at $x = a/2$, $y = b/2$. A rectangular protrusion in the pipe occupies the region $b-h<y<b$ and $|z|<w/2$ with $h<b/2$.

For very short bunches one can use the optical model to calculate the wakefield of the beam. The transverse impedance Z_y according to this model applied to the geometry of Fig. 1 is calculated in Appendix A and is given by Eq. (A.9). According to the optical model $Z_y \propto 1/\omega$, so that the product ωZ_y is independent of frequency. In this case the transverse wake of the bunch can be expressed through the transverse impedance of a point charge as follows:

$$W_y(z) = \frac{i}{2\pi}\int_{-\infty}^{\infty} Z_y(\omega)\tilde\lambda_z(\omega)e^{-i\omega z/c}\,d\omega = (\omega Z_y)\int_z^{\infty}\lambda_z(z')\,dz' \tag{9}$$

where $\tilde\lambda_z(\omega)$ is the Fourier transform of the longitudinal distribution $\lambda_z(z)$ ($\lambda_z(z)$ is normalized so that $\int\lambda_z(z)\,dz = 1$). Using (9) and assuming a Gaussian distribution function $\lambda_z(z)$ it is easy to find

$$\theta_{\rm rms} = \frac{Qe(\omega Z_y)}{\sqrt{12}\gamma mc^2}. \tag{10}$$

As is pointed out in Appendix A, for small values of h one can use the following formula for (ωZ_y):

$$\omega Z_y \approx \frac{8\pi^2 h}{a^2}F\left(\frac{b}{a}\right) \tag{11}$$

with the function F given by Eq. (A.11). Eqs. (10) and (11) allow one to evaluate the strength of the transverse kick for a given size of protrusion and to estimate the emittance increase of the beam. In the next section we will compare the wakefield predicted by the optical model with that obtained numerically using the 3D time domain code GdfidL and finally estimate the emittance growth for the parameters of the SuperKEKB collider.

4. SuperKEKB low energy ring

In this section we will estimate the effect of a single protrusion on the beam emittance for the parameters of the low-energy ring

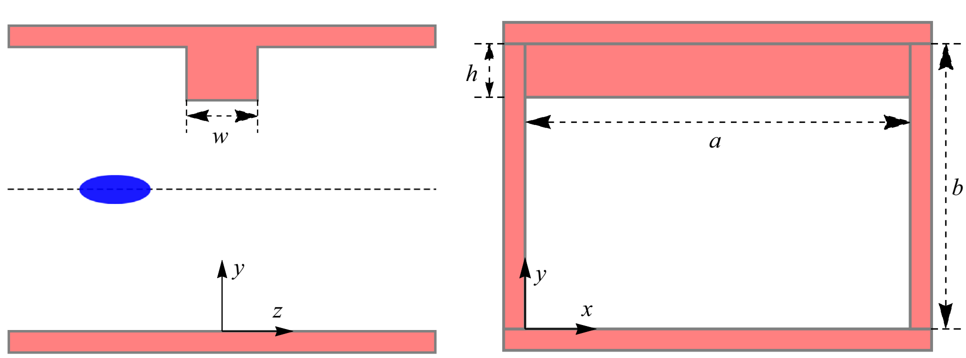

Fig. 1. Geometry of a rectangular vacuum chamber with an asymmetric protrusion. The design orbit goes through the center of the $a\times b$ cross-section.