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Beam-size-free optics determination

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ABSTRACT

A new method to measure the Twiss parameters in a beam transport line is presented. Usually these parameters are obtained based on the measured beam sizes. In the new method, in contrast, we determine them by finding quadrupole strengths that result in minimum beam sizes at a downstream measurement location. Therefore, systematic errors related to beam-size monitors do not propagate to the measured Twiss parameters. We describe the method together with a detailed estimation of statistical and systematic errors. It was examined with beam at the SwissFEL injector test facility at PSI, and these results are also presented.

1. Introduction

Most accelerator facilities have beam transport lines, where it is of importance to match the beam envelope to the design optics to enable their functionality. A beam transport line between an injector and a main ring, for example, is designed to transport the beam with a desired beam shape at the injection point. A linear accelerator itself is a beam transport line with accelerating structures, and it requires a specified beam envelope at some locations such as a beam collimator section.

In a beam transport line, the beam envelope is determined not only by the magnetic field but also by the phase space distribution of the incoming beam, and thus it is necessary to match the envelope based on beam measurements. Twiss parameters are usually obtained by measuring the beam sizes for various phase advances from the initial location to the measurement location. A variation of phase advance is introduced by changing the focusing between the two locations, i.e. the strength of single [1] or multiple [2] quadrupoles. These methods are often referred to as *quadrupole scans*. Alternatively, the beam-size measurement may be performed at several locations along the beam line [3]. All these methods rely on the measured absolute beam size to find the Twiss parameters together with the beam emittance, or at least relative beam sizes are used when only Twiss parameters are of interest.

Beam-size monitors, however, may suffer from systematic errors such as calibration errors or offsets which propagate to the final measurement results. We present a new method where specific quadrupole strengths are measured and employed to determine the Twiss parameters, avoiding systematic errors related to direct beam-size measurement. Calibration errors in the transfer functions of quadrupole magnets (quadrupole strength as a function of excitation current) still remain as a possible source of systematic errors. It is noted, however, that existing methods also suffer from such errors. To summarize our new method of beam optics determination is robust against systematic uncertainties in beam-size measurements and does not depend on the absolute value of the measured beam size. Because of the latter property, the method is called *beam-size-free optics determination*.

Our method relinquishes the direct measurement of the beam emittance. In a routine operation of a beam transport line, the emittance measurement may not be of interest because it is determined by the incoming beam. On the other hand, the beam matching may be performed when the measured Twiss parameters of the incoming beam differ from the design values. Therefore, it would make sense to focus and improve on the Twiss parameter measurement. The accuracy required for the Twiss parameter measurement may vary between accelerators. For example, an injection line into an electron storage ring would be rather relaxed just to avoid/minimize beam losses at the injection septum and other places while a fast injection line into a proton ring may require the highest possible accuracy to minimize emittance growth. At the same time, the achievable accuracy depends on the hardware involved in the measurement. We therefore performed numerical studies to quantify the Twiss parameter errors arising from various error sources. These results may allow a potential user of the method to judge if it is suitable for a particular case.

The principle of the method and an analytical estimation of the errors are described in Section 2, and numerical studies follow in

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Section 3. Measurement results obtained at the SwissFEL injector test facility are presented in Section 4. Finally the conclusions are drawn in Section 5.

2. Beam-size-free optics determination

2.1. Principle

The idea of the method is to determine the Twiss parameters from specific quadrupole strengths. Particularly, when the beam size is minimized in one plane at a location by varying the strength of a quadrupole, the Twiss parameter alpha of the plane at the entrance of the quadrupole can be represented as a function of the beta at the same location. Therefore the Twiss parameters are determined if two such relations are known. The quadrupole strength corresponding to the minimum beam size may be found from a beam-size scan, i.e. a set of beam-size measurements for various quadrupole strengths.

The propagation of the Twiss parameters between two locations is generally represented as

$$\begin{pmatrix} \beta_1 & -\alpha_1 \\ -\alpha_1 & \gamma_1 \end{pmatrix} = R \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix} R^T \tag{1}$$

where α, β and γ are the Twiss parameters and R is the (horizontal) transfer matrix from the location 0 to 1. The Twiss parameters satisfy

$$\gamma = \frac{1+\alpha^2}{\beta}. \tag{2}$$

From Eq. (1) we get

$$\beta_1 = R_{11}^2\beta_0 - 2R_{11}R_{12}\alpha_0 + R_{12}^2\gamma_0. \tag{3}$$

The location 0, for our purpose, may correspond to the entrance of the quadrupole used for the minimum beam-size scan and the location 1 corresponds to the beam-size measurement location. The normalized quadrupole strength (the integrated gradient normalized to the magnetic rigidity of the beam), K, is then contained in R_{11} but not in R_{12}. In cases where other quadrupoles between these two locations are turned off,

$$R = \begin{pmatrix} 1-Kl & l \\ -K & 1 \end{pmatrix} \tag{4}$$

where l is the distance from the location 0 to 1, and the thin-lens approximation for quadrupole is assumed without any loss of generality as discussed later. Therefore β_1 is a parabolic function of the quadrupole strength, and it is minimized when K is set to satisfy

$$\alpha_0 = \frac{R_{11}}{R_{12}}\beta_0. \tag{5}$$

Eq. (5) indicates that possible combinations of Twiss parameters at the entrance of the quadrupole correspond to a line in the alpha–beta space.

To determine the Twiss parameters at the initial location, we need a second beam-size scan with another quadrupole. Finally four beam-size scans will allow us to find the Twiss parameters in the horizontal and vertical planes.

The measurement is illustrated in Fig. 1. The Twiss parameters at the initial location i are to be measured. The two quadrupoles, Q_u (upstream-quad) and Q_d (downstream-quad), are used for the beam-size scans, and the beam size is always measured at the location of a monitor s. After the measurement, the beam will be matched either with further upstream quadrupoles to realize the design Twiss parameters at the location i or with quadrupoles

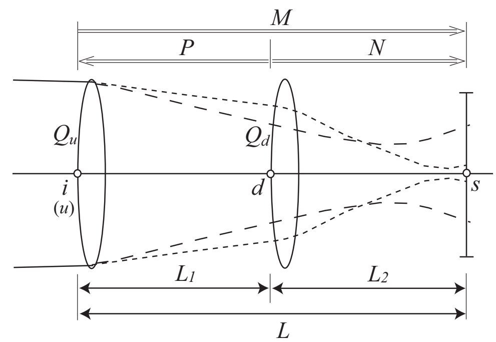

Fig. 1. Illustration of the measurement. The location i (or u) indicates the entrance of the upstream quadrupole Q_u, d the entrance of the upstream quadrupole Q_d, and s the beam-size measurement location. M is the transfer matrix from i to s, N is the transfer matrix from d to s, and P is the backward transfer matrix from d to i. L is the distance from i to s, L_1 from i to d, and L_2 from d to s. The dashed and dotted lines depict the beam envelope minimized at s by Q_u and Q_d, respectively. It is noted that the beam waist ($\alpha = 0$) appears upstream of the measurement location when the beam size is minimized.

downstream of the location i to let the beam follow the design optics afterward.

The downstream-quad is turned off during the upstream-quad scan while the strength of the upstream-quad for the downstream-quad scan is an important parameter. This is further discussed in Section 2.2.

Two beam-size scans for the horizontal plane give

$$\alpha_i = \frac{M_{11}}{M_{12}}\beta_i \tag{6}$$

and

$$\alpha_d^d = \frac{N_{11}}{N_{12}}\beta_d^d \tag{7}$$

where the subscripts i $(=u)$ and d refer to the locations shown in Fig. 1, and the superscript d indicates *downstream-quad* scan. M and N are the transfer matrices as in Fig. 1. According to this notation, β_d^d, for example, is the beta function at the location of d (subscript) when the downstream-quad scan is performed (superscript). Therefore, α_d^d and β_d^d depend on the strength of the upstream-quad, K_u^d. This notation applies to the following equations as well.

The slopes in the alpha–beta space are explicitly written as

$$\frac{\alpha_i}{\beta_i} = \frac{M_{11}}{M_{12}} = \frac{1}{L} - K_u^u \equiv F_u^u \tag{8}$$

and

$$\frac{\alpha_d^d}{\beta_d^d} = \frac{N_{11}}{N_{12}} = \frac{1}{L_2} - K_d^d \equiv F_d^d \tag{9}$$

where L_2 is the distance between the downstream-quad and the monitor (see Fig. 1).

In these equations, quadrupoles not involved in the measurement are omitted as being turned off for simplicity. Additional quadrupole(s) may, however, be useful. Since the method requires focusing of the beam in one plane, the beam size in the other plane can become so large that beam-size measurements may be spoiled. This can be avoided by turning on additional quadrupole (s) to focus the beam in the other plane. It is straightforward to show that F_u^u and F_d^d are uniquely determined even when several quadrupoles between the scanning quadrupole and the measurement location are turned on.