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The point spread function of electrons in a magnetic field, and the decay of the free neutron

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ABSTRACT

Experiments in nuclear and particle physics often use magnetic fields to guide charged reaction products to a detector. Due to their gyration in the guide field, the particles hit the detector within an area that can be considerably larger than the diameter of the source where the particles are produced. This blurring of the image of the particle source on the detector surface is described by a suitable point spread function (PSF), which is defined as the image of a point source. We derive simple analytical expressions for such "magnetic" PSFs, valid for any angular distribution of the emitted particles that can be developed in Legendre polynomials. We investigate this rather general problem in the context of neutron β-decay spectrometers and study the effect of limited detector size on measured neutron decay correlation parameters. To our surprise, insufficient detector size does not affect the accuracy of such measurements much, even for rather large radii of gyration. This finding can considerably simplify the layout of the respective spectrometers.

1. Introduction

In nuclear spectroscopy magnetic fields are frequently used to guide charged particles onto their detectors. This technique permits the use of relatively small detectors for a full 2π solid angle of detection, and even for 4π solid angle if two detectors are installed on either side of the source. In particle physics similar arrangements are found in time projection chambers. Such an arrangement is schematically shown in Fig. 1. The two detectors are then magnetically coupled not only to the source, but also to each other, which allows detecting particles in one detector that were backscattered on the other detector.

In particular, most modern spectrometers for the study of neutron decay parameters use magnetic fields to guide the decay electrons and/or protons onto their detectors. Examples are experiments or projects on neutron decay correlations, performed either with cold neutrons "in-beam" (abBA/PANDA [1], aCORN [2], aSPECT [3], Nab [4], PERC [5], Perkeo [6,7], Perkino [8], the latter with an artificial β-source, and Petersburg [9]) or performed with stored ultracold neutrons (UCNA [10,11], UCNB and UCNb [12]), as well as experiments on the neutron lifetime, in-beam (NIST [13]) or with stored UCN (HOPE [14], Mainz [15], PENeLOPE [16], UCNτ [17,18]). They all aim at relative accuracies of 10^{-3} or even 10^{-4}. In favorable cases measurements can be entirely free of detector edge effects, provided the detectors are sufficiently large to intercept particles up to the largest radii of gyration. The data produced are used in various fields of nuclear and astrophysics, as well as for sensitive tests of the standard model of particle physics, for details see the reviews [19–21].

In addition to higher count rates, the magnetic coupling of the source to the detectors has another advantage that had already been pointed out in Lee and Yang's seminal paper [22] on the parity violation in weak interactions. In experiments on the weak decay of polarized nuclei, a magnetic guide field permits a clean cut between those particles emitted into one half space under angles $\theta < \pi/2$, with respect to the local field direction, and those emitted into the other half space under $\theta > \pi/2$. This makes the measurement of correlation coefficients like the β-asymmetry independent of the precise direction of the local magnetic field and of the precise positioning of the detectors.

Evidently, the particle detectors must be large enough to accept all events that contribute significantly to the signal. To be safe one would tend to add, all around the image of the source on the detector of width $2x_n$, an area that allows detecting all incoming decay particles up to their maximum diameter of gyration $2r_{\max}$. The detector then must have a width of $2x_{\det} \geq 2x_n + 4r_{\max}$, as indicated in Fig. 1, and similarly for the height $2y_{\det}$ of the detector. If some fraction of these particles is backscattered on one detector,

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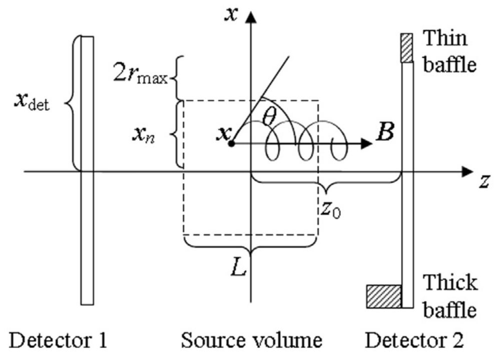

Fig. 1. A magnetic field B guides charged particles from a source volume of length L and width $2x_n$ to square detectors of width $2x_{det}$. The helical trajectories of the particles with helix angle θ have diameters of up to $2r_{max}$. Hence the particles reach the detector over a square area of width $2x_n+4r_{max}$. For the baffles see Section 3.4.

a new helical trajectory is started that may require an additional safety margin of $2r_{max}$ around the area of detection.

In Section 2 we first derive the distribution of charged particles, originating from a point source, on the surface of the detector. In imaging theory such a distribution is called a *point spread function* or PSF. In our case we regard a beam-optical imaging system, consisting of a point source at $\boldsymbol{x}=0$, and a uniform magnetic guide field along z that projects the charged particles onto the detectors installed at distances $\pm z_0$. We shall call the particle distribution function on the detector surface the *magnetic PSF*. We derive these magnetic PSFs analytically, and find surprisingly simple results for isotropic emission, for parity-violating asymmetric emission, and more generally for anisotropic charged particle emission of Legendre type, with the central result given in Eq. (25). There exist also analytical PSFs for the more general associated Legendre polynomials, as well as for the case of electron transport in non-uniform B-fields, but we postpone their discussion to a forthcoming publication.

In Section 3 we apply these PSFs to finite source volumes with position dependent source strength, and use the results for the ongoing neutron decay experiment PerkeoIII where $2r_{max}$ and x_n are of similar magnitude. In particular, we are interested to know how insufficient detector size will influence the results on neutron decay parameters.

The study of this problem seems to be a rather elementary exercise. However, as we shall see, this investigation requires some care, and the problem can be solved only using some subtle but very precise approximations, with a result that is both unexpected and comforting. At the same time we want to demonstrate how far one can go analytically before starting Monte Carlo simulations, which are difficult when investigating 10^{-4} effects for varying geometries. Charged particle guidance in a magnetic field is a widespread technique not only in nuclear physics, hence our results for the magnetic point spread functions may be of interest to a wider community.

2. The magnetic point spread functions

To be specific, we derive the magnetic point spread functions for the case of electron emission, although the results are valid for all kinds of charged particles. As an electron source we choose, again without loss of generality, free neutrons, instable against β-decay.

2.1. Source brightness

The exponential decay of a number of N_n neutrons in an active decay volume generates a flux of electrons

$$\Phi_{e0} = -\dot{N}_n = N_n/\tau_n \tag{1}$$

with the neutron lifetime $\tau_n=880$ s. A volume element d^3x located at position $\boldsymbol{x}$ gives rise to the local electron flux element $\phi_{e0}(\boldsymbol{x})=d\Phi_{e0}(\boldsymbol{x})=dN_n(\boldsymbol{x})/\tau_n$, with neutron density $\rho_n(\boldsymbol{x})$, or

$$\phi_{e0}(\boldsymbol{x}) = \rho_n(\boldsymbol{x})d^3\boldsymbol{x}/\tau_n. \tag{2}$$

The subscript "0" indicates that the flux is that at the source. The local brightness of the electron source then is defined as

$$b_{e0}(\boldsymbol{x}) = \frac{\partial^2\phi_{e0}(\boldsymbol{x})}{\partial E\ \partial\Omega} \tag{3}$$

with electron kinetic energy E and solid angle Ω of electron emission.

2.2. Electron trajectories

Let the electrons be guided by the B-field towards one of the two detectors. For a uniform field the guiding center (Fig. 2) is a straight line along z. For emission under polar angle θ with respect to the field direction z (Fig. 1), the radius of gyration of the electrons' helical trajectory is

$$r(E,\theta) = r_0(E)\sin\ \theta \tag{4}$$

with $0\le\theta\le\pi$ and

$$r_0(E) = p/eB = \sqrt{E(E+2m)}/eB \tag{5}$$

with electron charge e, mass m, and momentum p, and with light velocity $c\equiv 1$, see for instance Reference [23].

From the source to the detector, for each complete cycle, the electron progresses along z by the pitch of the helix,

$$d\ =\ 2\pi r_0\ \cos\ \theta. \tag{6}$$

With increasing helix angle θ, the pitch shortens and the diameter $2r$ of the helix widens, as shown in Fig. 3, which displays the relation $d^2+C^2=(2\pi r_0)^2$ between pitch d and circumference $C=2\pi r$ of the helix.

Upon arrival of the electron at the position z_0 of the detector, the total number of cycles is

$$n' = z_0/d \tag{7}$$

(with n' generally not an integer). The total phase angle α reached at z_0 is determined by the starting angle θ via

$$\alpha = 2\pi n' = z_0/(r_0\ \cos\ \theta). \tag{8}$$

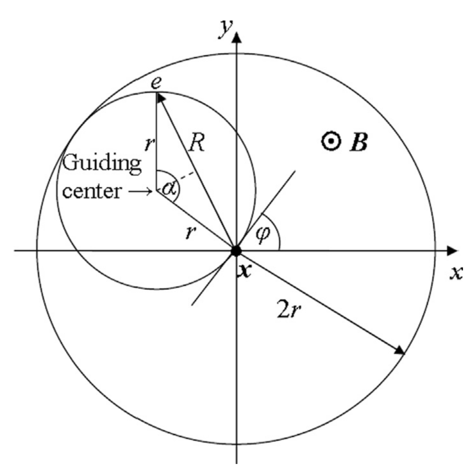

Fig. 2. An electron e, emitted in a uniform field B from a point source at position $\boldsymbol{x}$ under azimuth angle φ and polar or helix angle θ (the latter shown in Fig. 1), moves with gyration radius r along a helical path about the B-field. The axis of the helix is along z (at right angles to the paper plane) and is called the guiding center. After spiraling through total phase angle α, the electron reaches the detector surface a distance R away from the "projected" source position $\boldsymbol{x}$. The circle of radius $2r$ indicates the reach of electrons with varying φ.