Composites: Part B 62 (2014) 54–64

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Mechanical and thermal buckling analysis of functionally graded plates resting on elastic foundations: An assessment of a simple refined nth-order shear deformation theory

Hessameddin Yaghoobi *, Abdolhossein Fereidoon

Faculty of Mechanical Engineering, Semnan University, Semnan, Iran

ARTICLE INFO

Article history:
Received 30 September 2013
Received in revised form 17 January 2014
Accepted 14 February 2014
Available online 24 February 2014

Keywords:
B. Buckling
A. Plates
C. Analytical modeling
C. Computational modeling

ABSTRACT

In present study, a refined nth-order shear deformation theory is proposed, formulated and validated for a variety of numerical examples of functionally graded (FG) plates resting on elastic foundation for the mechanical and thermal buckling responses. The present refined nth-order shear deformation theory is based on assumption that the in-plane and transverse displacements consist of bending and shear components, in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. The most interesting feature of this theory is that it accounts for a parabolic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. Governing equations are derived from the principle of minimum total potential energy. A Navier type closed form solution methodology is also proposed for simply supported FG plates resting on elastic foundation which provides accurate solution. The accuracy of the present theory is verified by comparing the obtained results with those predicted by classical plate theory (CPT), first-order shear deformation theory (FSDT), higher-order shear deformation theory (HSDT) and refined plate theory (RPT). Moreover, results show that the present theory can achieve the same accuracy of the existing higher-order shear deformation theories which have more number of unknowns.

1. Introduction

Due to high performance heat resistant capacity and excellent characteristics in comparison with conventional composites, Functionally Graded Materials (FGMs) have attracted considerable attention recent years. FGMs are microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. They are usually made from a mixture of ceramics and metals to attain the significant requirement of material properties. Buckling behavior of functionally graded (FG) structures under different types of loading is important for practical applications and has received considerable interest.

Javaheri and Eslami [1,2] investigated buckling analysis of functionally graded plates (FGPs) under four types of thermal loads based on the classical plate theory (CPT) and the higher order shear deformation plate theory (HSDT), respectively. Lanhe [3] analytically studied the thermal buckling problem of a functionally graded plate (FGP) with moderately thickness and simply supported boundary conditions based on the first order shear deformation theory (FSDT). Samsam Shariat and Eslami [4] analyzed both the thermal and mechanical buckling of imperfect FGPs with simply supported boundary conditions on the basis of the CPT. Matsunaga [5] developed a two-dimensional global higher-order deformation theory for thermal buckling of plates made of FGMs. He calculated the critical buckling temperatures of a simply supported FGP subjected to uniformly and linearly distributed temperatures. Zhao et al. [6] investigated the buckling behavior of FGPs under mechanical and thermal loads with arbitrary geometry, including plates that contain square and circular holes at the center, is investigated using the element-free *kp*-Ritz method. Zenkour and Sobhy [7] studied the critical buckling temperature for FGM sandwich plates. They used sinusoidal shear deformation plate theory to deduce the stability equations. Tung and Duc [8] investigated buckling of thick FGPs with initial geometrical imperfection under thermal loadings. By Galerkin method, the resulting equations were solved to obtain closed-form solutions of critical buckling temperature difference. Bodaghi and Saidi [9] presented a benchmark solution for the critical buckling temperatures of thick

* Corresponding author.
E-mail address: Yaghoobi.Hessam@gmail.com (H. Yaghoobi).

http://dx.doi.org/10.1016/j.compositesb.2014.02.014
1359-8368/

functionally graded rectangular plates with various boundary conditions. Recently, the buckling of heated FGM annular plates based on the CPT was studied analytically by Kiani and Eslami [10]. An exact analytical solution was presented to calculate the thermal buckling load by obtaining the eigenvalues of the stability equation. Neves et al. [11] derived a HSDT for modeling FGPs accounting for extensibility in the thickness direction. They analyzed static, free vibration and buckling bahavior of isotropic and sandwich FGPs.

The components of structures widely used in aircraft, reusable space transportation vehicles and civil engineering are usually supported by an elastic foundation. Therefore, it is necessary to account for effects of elastic foundation for a better understanding of the buckling behavior of plates. To describe the interactions of the plate and foundation there are various kinds of foundation models. The simplest model for the elastic foundation is Winkler or one-parameter model [12], which regards the foundation as a series of separated springs without coupling effects between each other. Pasternak [13] improved this model by adding a shear layer to Winkler model. Pasternak or two-parameter model is widely used to describe the mechanical behavior of structure–foundation interactions. In spite of practical importance and increasing use of FG structures, investigation on buckling of FG plates supported by elastic media is limited in number [14–26].

The problem of the choice of the theories used in the expansion of the different variables is crucial to adequately approximate the real behavior of a given structure. In the past few decades, various shear deformation theories have been proposed and implemented to analysis of plates. Moreover, increased use of advanced materials in primary structures necessitates the development of precise theoretical model to accurately predict the behavior of the structures. The researchers have paid much attention for modeling of the plates over the past few decades and a variety of plate theories have been introduced. The CPT can only provide reasonable results for thin plates since it disregards the effects of the transverse shear deformation and transverse normal stress. The FSDT [27,28] considers the effect of transverse shear deformation, but this theory needs a shear correction factor in order to satisfy zero transverse shear stress boundary conditions on the top and bottom. The various higher order shear deformation theories (HSDTs) were proposed to analyze the plates by Ambartsumian [29], Levinson [30], Touratier [31], Soldatos [32], Karama et al. [33], Aydogdu [34], Reddy [35], Xiang et al. [36] and Mantari et al. [37]. On the other hand, Shimpi [38] has developed a new refined plate theory (RPT) which is simple to use. The refined theory proposed by Shimpi is based on the assumption that the in-plane and transverse displacements consist of bending and shear components in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. The most interesting feature of this theory is that it accounts for a quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. In addition, it has strong similarities with the CPT in some aspects such as governing equation, boundary conditions and moment expressions.

The present paper deals with the nth-order shear deformation theory [36]. The effectiveness and accuracy of this theory is demonstrated by Xiang et al. [39–42]. Moreover, the present paper mainly uses the ideas behind the new refined plate theory [38] that the authors include w_b and w_s (bending and shear transverse displacement) to model the transverse displacement of the shear deformation theories (in many theories assumed constant and called w_0 [36–42]). In the present paper, the authors combine this idea for developing the nth-order shear deformation theory with modified displacement field to its optimization. The present theory has only four unknowns and four governing equations, but it satisfies the stress-free boundary conditions on the top and bottom surfaces of the plate without requiring any shear correction factors. The displacement field of the proposed theory is chosen based on a constant transverse displacement and nth-order variation of in-plane displacements through the thickness. The partition of the transverse displacement into the bending and shear parts leads to a reduction in the number of unknowns and governing stability equations of nth-order shear deformation theory, hence makes the theory simple to use. Indeed, the number of unknown functions involved in the present theory is only four, as opposed to five in the case of other shear and normal deformation theories. Governing equations are derived from the principle of minimum total potential energy. Closed form solutions for mechanical and thermal buckling analysis of FG plates resting on elastic foundations are obtained. Numerical examples are presented to verify the accuracy of the present theory. The present refined nth-order shear deformation theory is reported for the first time and can be served as benchmark results for researchers to validate their theories in the future.

2. Material properties of FG plate

In this study, material properties of a FG plate are considered to vary in accordance with the rule of mixtures as [43]. Simple power law distribution from pure metal at bottom face ($z = -h/2$) to pure ceramic at the top face ($z = +h/2$) in terms of the volume fractions of the constituents is assumed [44]. The mechanical and thermal properties of FGMs are determined from the volume fraction of the material constituents. We assume that the material properties such as the modulus of elasticity (E), the thermal conductivity (K), coefficient of thermal expansion (α) and Poisson's ratio (υ) can be determined by [24,25,45,46]:

$$E(z) = E_M + (E_C - E_M)\left(\frac{2z+h}{2h}\right)^k,$$
$$K(z) = K_M + (K_C - K_M)\left(\frac{2z+h}{2h}\right)^k$$
$$\alpha(z) = \alpha_M + (\alpha_C - \alpha_M)\left(\frac{2z+h}{2h}\right)^k, \quad \upsilon(z) = \upsilon = \text{constant} \tag{1}$$

where k is the gradient index and subscripts M and C refer to the metal and ceramic constituents, respectively. The value of k equal to zero and infinity represents a fully ceramic and metal plate, respectively.

3. New refined nth-order plate theory

3.1. Kinematics and constitutive equations

In this study, further simplifying assumptions are made to the nth-order shear deformation theory so that the number of unknowns is reduced. The displacement field of the conventional nth-order shear deformation theory is given by [36]

$$u_1(x,y,z) = u_0(x,y) + z\phi_x(x,y) - \frac{1}{n}\left(\frac{2}{h}\right)^{n-1} z^n\left(\phi_x(x,y) + \frac{\partial w_0(x,y)}{\partial x}\right) \tag{2a}$$

$$u_2(x,y,z) = v_0(x,y) + z\phi_y(x,y) - \frac{1}{n}\left(\frac{2}{h}\right)^{n-1} z^n\left(\phi_y(x,y) + \frac{\partial w_0(x,y)}{\partial y}\right)$$
$$n = 3, 5, 7, 9, \ldots \tag{2b}$$

$$u_3(x,y,z) = w_0(x,y) \tag{2c}$$