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Landau damping and the head–tail instability of the coupled synchro-betatron coherent oscillations

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ABSTRACT

Within the hollow-bunch approximation we study variations in the stability conditions of the coupled synchro-betatron coherent oscillations due to common effects of the lattice chromaticity and of the Landau damping of such oscillations. We assume that the Landau damping of coherent oscillations is provided by octupole fields of the ring lattice. We also assume that the wakefields of the bunch decay substantially during the revolution period of particles along the closed orbit. For this reason, the memory of the bunch wakefields is ignored in this paper.

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1. Introduction

In our recent paper [1] we calculated the stability diagrams for the coupled synchro-betatron coherent oscillations of a single bunch, which interacts with its surroundings via a wideband transverse impedance and which are suppressed due to the Landau damping by octupole fields of the ring lattice. In these calculations we focused on the effects of the shape of the betatron frequency distribution functions of the bunch on the shape of the stability diagrams of the coupled synchro-betatron coherent oscillations. For this reason, we ignored in Ref. [1] the effects of the lattice chromaticity on the stability of such coherent oscillations. Meanwhile, closer inspections show that the head–tail instabilities, or damping (see in Ref. [2], or in Ref. [3]), can change the stability conditions of the coupled synchro-betatron modes and, hence, can change the shape and the widths of their stability diagrams.

In this paper we study the variations in the stability diagrams for the coupled betatron and synchro-betatron collective modes of a single bunch due to common effects of the Landau damping and of the head–tail instability of these oscillations due to the lattice chromaticity. We assume that the Landau damping of coherent oscillations occurs due to octupole fields of the ring lattice.

2. General equations

Following the paper [1], we take that the canonical transformation to the action-phase variables of unperturbed oscillations of the bunch particles is generated using the formulae:

$$y = a_y \cos\phi_y, \quad p_y = -\omega_0 \nu_{y0} a_y \sin\phi_y,$$
$$x = a_x \cos\phi_x, \quad p_x = -\omega_0 \nu_{x0} a_x \sin\phi_x, \quad \frac{d\psi_{y,x}}{dt} = \omega_0 \nu_{y,x},$$
$$\theta = \omega_0 t + \phi, \quad \Delta p = p - p_0,$$
$$\frac{d\phi}{dt} = \omega_0 \eta \frac{\Delta p}{p_0}, \quad \phi = \varphi \cos\psi_s, \quad \frac{d\psi_s}{dt} = \omega_0 \nu_s,$$
$$\phi_{y,x} = \psi_{y,x} + \phi \frac{d\omega_{y,x}}{d\omega_0} = \psi_{y,x} + \phi\left(\nu_{y0,x0} + \frac{\xi_{y,x}}{\eta}\right), \quad \xi_{y,x} = \frac{d\nu_{y,x}}{d\ln p}$$
$$I_y = \frac{p_0 \nu_{y0} a_y^2}{2R_0}, \quad I_x = \frac{p_0 \nu_{x0} a_x^2}{2R_0}, \quad \eta = \frac{1}{\gamma^2} - \alpha \tag{1}$$

here the symbols y and x mark the values related to the vertical (y) and to the horizontal betatron oscillations of particles, $\Pi = 2\pi R_0$ is the perimeter of the closed orbit, the suffix 0 marks the values, calculated for the synchronous particle, $E_0 = \gamma M c^2$ is the particle energy, $\nu_{y,x,s}$ are respectively the tunes of the betatron and of the synchrotron oscillations of particles and α is the momentum compaction factor of the ring.

Due to nonlinear dependencies of the lattice focusing, or defocusing forces on the particle offsets from its position on the closed orbit, the frequencies $\omega_{y,x}$ and ω_s may depend on the amplitudes of the particle betatron and synchrotron oscillations. In this paper we simplify the calculations assuming that the bunch length is short enough to enable neglecting the nonlinearity of the incoherent synchrotron oscillation. It means that in our calculations we take that the frequency ω_s has the same value for all particles of the bunch. Concerning the frequencies of the betatron

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oscillations of the particles we assume that those linearly depend on the squares of the oscillation amplitudes $2R_0 I_{y,x}/(p\nu_{y0,x0})$. For example, for the vertical incoherent betatron oscillations we write

$$\omega_y = \omega_0\nu_{y0} + aI_y - bI_x \quad (2)$$

where the values a and b are determined by the strength of the lattice octupole field. According to this equation, the frequency of the vertical betatron oscillations depends on both variables $\omega_y = \omega_y(I_y, I_x)$. These dependences produce the so-called partial frequency spreads of the incoherent betatron oscillations in the bunch. Following the paper [1], we call the frequency spread $\omega_y(I_y, I_x = 0)$ as the own frequency spread for the vertical betatron oscillations. Correspondingly, we call the frequency spread $\omega_y(I_y = 0, I_x)$ as the external frequency spread for the vertical betatron oscillations. Variations in the partial frequency spreads, generally, change the distribution functions in the frequencies of the vertical betatron oscillations and, hence, can change the stability conditions of coherent oscillations of the bunch as well as the threshold number of particles in the bunch.

We consider the case of the vertical dipole betatron and synchro-betatron coherent oscillations. For simplicity we also assume that the interaction of the bunch with its surroundings can be described in terms of a wideband localized transverse coupling impedance and that the bunch wakefields completely decay during a single turn. As usual, we take that the bunch without coherent oscillations is described using the distribution function

$$f = \frac{f_0(I_y, I_x)\rho(\varphi)}{(2\pi)^3}. \quad (3)$$

We assume that the functions $f_0(I_y, I_x)$ and $\rho(\varphi)$ obey the following normalization conditions:

$$\int_0^\infty dI_x \int_0^\infty dI_y\, f_0(I_y, I_x) = 1, \quad \int_0^\infty d\varphi\, \varphi\rho(\varphi) = 1.$$

The vertical dipole coherent oscillations of the bunch are described by a small addition δf to f. Assuming a study of the stability condition problem, we write

$$f = \frac{f_0(I_y, I_x)\rho(\varphi)}{(2\pi)^3} + \sqrt{I_y}\frac{df_0}{dI_y}\sum_{m=-\infty}^{\infty}\chi_m(\varphi)\exp(im_y\psi_y + im\psi_s - i\omega t) \quad (4)$$

where $m_y = \pm 1$. If the perturbations of particle oscillations by the bunch wakes result in reasonable weak variations of amplitudes of coherent oscillations during the periods of incoherent betatron oscillations of particles, the linearized Vlasov equation for δf enables one to find out that the amplitudes χ_m obey the following system of homogeneous integral equations (see, e.g. in Ref. [4]):

$$\chi_m(\varphi) = \rho(\varphi)\int_{-\infty}^{\infty} dn\, \Omega_m(n) J_m(\varphi[n+\xi_1]) \times \sum_{m'=-\infty}^{\infty} F(\Delta\omega_m - m'\omega_s)\int_0^\infty d\varphi'\, \varphi' J_{m'}(\varphi'[n+\xi_1])\chi_{m'}(\varphi') \quad (5)$$

here $\xi_1 = m_y\xi_y/\eta$,

$$F(\omega) = -\int_0^\infty dI_x \int_0^\infty dI_y \frac{I_y(\partial f_0/\partial I_y)}{\Delta\omega_m - m_y(aI_y - bI_x)}, \quad \mathrm{Im}\,\omega > 0 \quad (6)$$

where $\Delta\omega_m = \omega - m_y\omega_0\nu_{y0}$ and $J_m(x)$ is the Bessel function. The function $\Omega_m(\omega)$ in Eq. (5) presents the value of the coherent frequency shift of a coasting beam which has the same number of particles as our bunch and which interacts with the same transverse coupling impedance

$$\Omega_m(\omega) = im_y\frac{Ne^2\omega_0}{4\pi p\nu_{y0}} Z_\perp(\omega). \quad (7)$$

Below we simplify the calculations assuming that

$$Z_\perp(\omega) = -\frac{R_0}{l_\perp^2}\frac{Z^\parallel(\omega)}{(\omega/\omega_0)} \quad (8)$$

where $l_\perp$ is the transverse distance from the closed orbit to the electrodes, $Z^\parallel(\omega)$ is the longitudinal coupling impedance of the bunch and electrodes.

In Eq. (5) we used an assumption that the bandwidth of the impedance substantially exceeds the revolution frequency of the bunch particles. For this reason, an exact value of the frequency in the argument of $\Omega_{m_y}(\omega)$ in Eq. (5) was replaced by a combination frequency from the unperturbed spectrum of coherent oscillations $\omega = \omega_0(n + m_y\nu_{y0})$, while the summation over discrete harmonics of the revolution frequency $n = \omega/\omega_0$ was replaced by the integration over continuous harmonic numbers (e.g. in Ref. [4]).

Eq. (5) describes coherent oscillations of a bunch with coupled betatron and synchro-betatron modes. It is clear that such a coupling will be strong only in cases, when the coherent frequency shift of the bunch $\Delta\omega_m$ becomes comparable or higher than the frequency of synchrotron oscillations of particles ω_s. Typically, these general integral equations are too complicated to enable their solution in an analytic, or in a numerical form. In order to avoid this embarrassment and to focus on the effect of the Landau damping and of the head–tail effect on the stability of the coupled synchro-betatron modes of the bunch, we consider a simplified model, where all particles of the bunch have equal amplitudes of the synchrotron oscillations

$$\rho(\varphi) = \delta\left(\frac{\varphi^2 - \varphi_0^2}{2}\right). \quad (9)$$

In this case and for relativistic energies of particles ($\gamma \gg 1$), the solutions to Eq. (5) read

$$\chi_m(\varphi) = C_m\delta(\varphi_0^2 - \varphi^2) \quad (10)$$

which replaces the system of integral equations in Eq. (5) by the infinite system of algebraic equations for amplitudes C_m

$$C_m = -\frac{im_yNe^2c}{4\pi p\nu_{y0}l_\perp^2}\int_{-\infty}^{\infty}\frac{dn\, Z^\parallel(n)}{n} J_m(n+\varphi_0\xi_1) \times \sum_{m'=-\infty}^{\infty} F(\Delta\omega_m - m'\omega_s) J_{m'}(n+\varphi_0\xi_1) C_{m'}. \quad (11)$$

Defining in these equations $\zeta = \varphi_0\xi_1$ and $x = \Delta\omega_m/\omega_s$, we obtain

$$C_m = w\sum_{m'=-\infty}^{\infty} B_{m,m'}F(x-m')C_{m'} \quad (12)$$

where[1]

$$w = \frac{m_yNe^2}{4\pi p\nu_{y0}l_\perp^2\,\omega_s} \quad (13)$$

and

$$B_{m,m'} = c\int_{-\infty}^{\infty}\left(\frac{-iZ^\parallel(n)}{n}\right) J_m(n+\zeta) J_{m'}(n+\zeta)\, dn \quad (14)$$

$$F(x-m) = -\int_0^\infty dI_x\int_0^\infty dI_y\frac{I_y(\partial f_0/\partial I_y)}{x - m - m_y(aI_y - bI_x)/\omega_s}. \quad (15)$$

In Eq. (12) the effects of the Landau damping are described by the factors $F(x-m')$, while the head–tail instabilities are described by the matrix elements $B_{m,m'}$. This factorization of the matrix in Eq. (12) can substantially simplify numerical calculations with this

[1] For a bunch in a strong focusing storage ring the factor $1/\nu_{y0}$ in Eq. (13) should be replaced by the ratio β_{y0}/R_0, where β_{y0} is the value of the β-function at the location of the coupling impedance (e.g. in Ref. [4]).

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