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Symmetric single-quadrupole-magnet scan method to measure the 2D transverse beam parameters

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ABSTRACT

Precise measurements of the transverse beam parameters are essential to control and optimize all types of charged particle beams. In this paper we present a novel method that uses one quadrupole magnet and one profile monitor to measure the transverse beam emittance and optics. In comparison to a conventional single-quadrupole scan measurement, this new technique measures the two transverse planes simultaneously. This novel procedure is faster, more intuitive and allows keeping under control the required quadrupole gradient and the beam sizes at the profile monitor. The application of the method is illustrated with the SwissFEL Injector Test Facility.

1. Introduction

An accurate knowledge of the transverse beam properties is fundamental in all types of charged particle accelerators. For instance, in Free-Electron Lasers (FEL) a high quality electron beam with low emittance and matched optics is required for the optimal lasing process.

The 2-dimensional (2D) transverse beam matrix composed of the second-order moments of the beam distribution describes the statistical properties of the beam:

$$\sigma_{uu} = \begin{pmatrix} \langle u^2 \rangle & \langle uu' \rangle \\ \langle uu' \rangle & \langle u'^2 \rangle \end{pmatrix} \quad (1)$$

where u refers to either x (horizontal coordinate) or y (vertical coordinate), and u' is the derivative of u with respect to the longitudinal coordinate. The beam emittance and optical functions (α, β and γ, also called Twiss parameters) can be derived as follows:

$$\varepsilon_u = \sqrt{\det(\sigma_{uu})} \quad (2)$$

$$\alpha_u = -\langle uu' \rangle / \varepsilon_u, \quad \beta_u = \langle u^2 \rangle / \varepsilon_u, \quad \gamma_u = \langle u'^2 \rangle / \varepsilon_u. \quad (3)$$

When we multiply the emittance by the beam total momentum we get the so-called *normalized* emittance:

$$\varepsilon_{N_u} = \frac{p}{mc} \varepsilon_u \quad (4)$$

where p is the central momentum of the beam, m is the particle mass and c is the speed of light.

The transport of the 2D beam matrix from s_0 to s can be described by a matrix formalism [1,2]:

$$\sigma_{uu}(s) = R \cdot \sigma_{uu}(s_0) \cdot R^T \quad (5)$$

where R is the transverse transfer matrix from s_0 to s:

$$R = \begin{pmatrix} R_{uu} & R_{uu'} \\ R_{u'u} & R_{u'u'} \end{pmatrix}. \quad (6)$$

According to Eqs. (1), (5) and (6), the beam size at a position s can be expressed as follows:

$$\langle u^2 \rangle_s = R_{uu}^2 \langle u^2 \rangle_{s_0} + R_{uu'}^2 \langle u'^2 \rangle_{s_0} + 2R_{uu} R_{uu'} \langle uu' \rangle_{s_0}. \quad (7)$$

The elements of σ_{uu} can be reconstructed at s_0 by measuring the beam size at s for different optics transformations between s_0 and s. At least three measurements (i.e. three equations) are needed to reconstruct the three second moments of the beam, but more measurements will allow improving the robustness of the reconstruction. Ideally, the betatron phase-advance μ_u between the observation and reconstruction points should be scanned progressively between 0 and π for the best reconstruction of the 2D parameters.

In order to obtain the 2D parameters the beam sizes can be measured at different positions along a fixed lattice (multiple-position approach like FODO measurements) or alternatively the optics between the reconstruction and the observation points can be varied with one or more quadrupole magnets (multiple-optics approach or quadrupole scans). The multiple-optics approach has the inconvenience of modifying the optics during a measurement, but it is more compact, that requires less equipment and is more flexible than the multiple-position strategy. Because of that in

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quadrupole scans there is no need for a long diagnostic section, therefore the overall length of a typical accelerator is reduced.

Refs. [3–8] give more detailed information about definitions and measurement procedures concerning the 2D transverse beam parameters.

In a conventional single-quadrupole scan measurement the phase-advance is controlled only in one plane at a time. Moreover, the beam size can go through a very tight focus at the measurement position during the scan, which makes the reconstruction of the parameters very difficult and sensitive to beam size measurement errors. In addition, the beam size variations can be very large along the scan, which may give rise to dynamic range problems for the profile monitor.

This document presents a special single-quadrupole scan measurement procedure with the following features:

- The horizontal and vertical phase-advances are scanned simultaneously. This improves the time of the measurement by a factor of about two with respect to conventional measurements.
- The β-function at the measurement point is the same in x and y for any quadrupole gradient. Consequently the ratio between the beam size in x and y should be constant for the whole scan (i.e. round beam if the emittances are equal in both planes), but varies for mismatched beam parameters at the reconstruction point. It will be easy and intuitive to measure whether the beam is matched or not.
- The scan produces a gentle waist. The β-function at the waist is relatively large (with typical values of ~ 10 m) and the beam size variations at the measurement location are kept within moderate limits. Therefore, dynamic problems of the profile monitors and space charge effects are avoided.
- The range of the quadrupole gradient required for the measurement stays within reasonable limits, which minimizes possible chromatic effects and trajectory deviations during the scan.
- The β-function at the measurement position and the required quadrupole strengths can be tuned by changing some parameters, such as the covered phase-advance or the initial optics.

2. Description and formulas

The measurement set-up consists simply of a quadrupole, a drift space, and a profile monitor, which measures the beam sizes in both planes simultaneously. The quadrupole entrance is the reference point where the beam parameters (emittance and optical functions) are reconstructed. All the equations presented in this section are obtained assuming thin-lens approximation for the quadrupole magnet.

The optics at the reconstruction point need to be identical in both planes if we want to simultaneously scan x and y with a single quadrupole. We define the optics at the reconstruction position as follows:

$$\alpha_x(s_0) = \alpha_y(s_0) = \alpha_0$$
$$\beta_x(s_0) = \beta_y(s_0) = \beta_0$$
$$\gamma_x(s_0) = \gamma_y(s_0) = \gamma_0. \tag{8}$$

As exemplified in Fig. 1, the propagated β-functions downstream of the quadrupole cross each other at a given point only if the beam is initially convergent. This happens for any quadrupole gradient at a distance L from the quadrupole such that:

$$\alpha_0 \cdot L = \beta_0. \tag{9}$$

If this condition is fulfilled the sum of the phase-advances in x and y (between the quadrupole and the profile monitor) will be

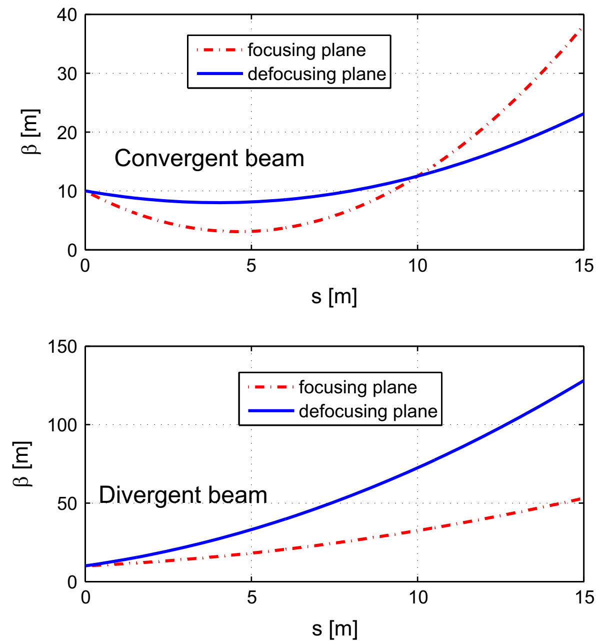

Fig. 1. β-function along a drift after a quadrupole with a certain strength. The upper plot shows a case where the initial beam is convergent ($\beta_0 = 10$ m, $\alpha_0 = 1$) and the lower plot corresponds to a divergent initial beam ($\beta_0 = 10$ m, $\alpha_0 = -1$). Only if the beam is initially convergent there is a position where the β-functions are the same in both planes. This point is at a distance $L = \beta_0/\alpha_0$ from the quadrupole (in our example $L = 10$ m). This condition holds for any gradient of the quadrupole magnet and also satisfies $\mu_x + \mu_y = \pi$.

exactly 180°:

$$\mu_x + \mu_y = \pi. \tag{10}$$

This means that the scan will be equivalent in x and y, since for the reconstruction of the 2D parameters a certain phase-advance μ provides the same information as $\pi - \mu$.

The integrated focusing of the quadrupole can be obtained as a function of the phase-advance and the initial β-function using the following equation:

$$\frac{1}{f} = kl = -\frac{1}{\tan\,\mu_x \cdot \beta_0} = \frac{1}{\tan\,\mu_y \cdot \beta_0}. \tag{11}$$

One can see that for no quadrupole gradient the phase-advance in x and y is identical and equal to $\pi/2$. The range of the quadrupole gradients to perform the scan will depend on the phase-advance that we want to cover. For example, for $\beta_0 = 10$ m we will need $kl = \pm 0.10\ \mathrm{m}^{-1}$ to cover $\pi/2$ or $kl = \pm 0.24\ \mathrm{m}^{-1}$ to scan $3\pi/4$.

The β-function at the measurement point can be obtained as follows:

$$\beta = \frac{L^2}{\beta_0 \cdot \sin^2 \mu} \tag{12}$$

where μ is the phase-advance in x or y. The observed minimum β-function at the measurement location occurs for no quadrupole gradient, when $\mu_x = \mu_y = \pi/2$:

$$\beta_{\min} = \frac{L^2}{\beta_0}. \tag{13}$$

This quantity can be controlled by changing the initial optics and the distance between the quadrupole and the profile monitor. For our example where $\beta_0 = 10$ m and $L = 10$ m, $\beta_{\min}$ will also be 10 m.

From Eqs. (12) and (13) it can be seen that the ratio between the β-function and the $\beta_{\min}$-function only depends on the covered