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Analytical evaluations of coupling impedances of resistive and magnetic bellows

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ABSTRACT

A theory is developed to calculate both the longitudinal and transverse impedances of magnetic as well as resistive bellows with cylindrical symmetry that is sandwiched between chambers with perfectly conductive metal fittings. Analytical estimations of the impedances are necessary because the skin depth is too small to make sufficiently tiny mesh sizes in current numerical codes. The impedances of bellows made of materials having both large conductivity and permeability are drastically increased owing to magnetic effects, compared with those of bellows made of perfectly conductive materials.

1. Introduction

The 3 GeV rapid cycling synchrotron (RCS) in Japan Proton Accelerator Research Complex (J-PARC) [1] is one of the facilities [2,3] aiming at MW-class output beam power. At the RCS, the significant impacts of the fields from the extraction beam transport line to the circulation beams were found at the early beam commissioning stage [4]. In general, lattice imperfections break the super-periodicity of rings and excite undesirable resonances [5], which makes it difficult to generate high-intensity beams in the rings. Thus, Hotchi et al. have done the simulation studies and proposed to shield the leakage fields to perform high-intensity beams [6]. Accordingly, Kamiya et al. fabricated the chambers and bellows made of permalloy [7], which are now installed and are successfully shielding the field from the extraction beam transport line from leaking into the beam circulation area [8].

On the other hand, when the chambers are made of resistive materials, the resistive wall impedances are excited by a passage of the beam [9,10]. In the conventional formulae for the resistive wall impedance [9,10], the dielectric constant and the magnetic permeability are assumed to be those of vacuum, i.e., ε_0 and μ_0, respectively. However, when the chambers are made of materials where both magnetic permeability and conductivity are high, such as permalloy (typical values of the conductivity and the initial relative permeability of permalloy are 1.5×10^6–$1.8\times10^6/\Omega\,\mathrm{m}$ and 30 000–75 000, respectively [11]), the coupling impedance due to the magnetic permeability as well as the conductivity should be considered. Resistive wall impedances have been extensively studied by many researchers [12–14]. In those studies, the effects of both conductivity σ_c and relative permeability μ_r are included. To make matters worse, the longitudinal and transverse impedances are about $\sqrt{\mu_r}$ times larger than the conventional resistive wall impedance for thick chambers and relativistic beams.

Here arises the question of to what degree are the impedances of other structures made of resistive and magnetic materials enhanced, compared with structures made of perfectly conductive materials. In this study, we focus on the case of bellows made of permalloy.

Recently, numerical simulation codes have become practical, powerful tools to evaluate impedances [15,16]. When the bellows are made of perfectly conductive materials, the tools accurately estimate the coupling impedances. However, when the bellows are made of resistive materials, it is almost impossible to numerically evaluate them, because the skin depth is so small that a sufficiently tiny mesh size cannot be made in the codes. Accordingly, it is necessary to analytically investigate the coupling impedance of bellows made of materials having finite magnetic permeability as well as conductivity.

A pioneering study that analytically investigated the impedances of bellows made of perfectly conductive materials was conducted by Kheifets and Zotter [17]. They estimated the impedance of infinitely long bellows. In real accelerators, the bellows is sandwiched between chambers with metal fittings, and the radial size of the bellows is typically larger than the radius of the chamber. Consequently, a gap appears between the chambers and as does a space outside the gap. Recently, a theory of the impedance of a gap with azimuthal symmetry has been developed [18]. In this paper, the impedance of such three-dimensional bellows that is sandwiched between the chambers is analytically investigated by generalizing the theory of gap impedance. The effects of magnetic permeability and conductivity of the bellows are included as well.

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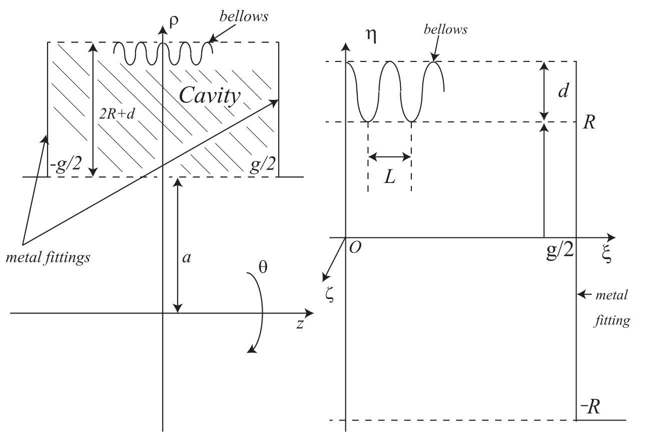

Fig. 1. A schematic picture of bellows sandwiched between chambers with metal fittings. The left figure shows the overall view using global coordinates (ρ, θ, z), and the right figure shows the scaled figure of the part of the cavity in local coordinates (ξ, η, ζ).

In Section 2, the theory is developed to obtain the expressions for both the longitudinal and transverse impedances, especially in the low-frequency region. The theoretical results and the numerical results obtained with the simulation code ABCI [15] are also compared in this section. The paper is summarized in Section 3.

2. Impedances of bellows

2.1. Configuration of bellows

Let us consider a cylindrically symmetric bellows. A schematic picture of the bellows that is sandwiched between chambers with metal fittings is shown in the left figure of Fig. 1. Cylindrical coordinates (ρ, θ, z) are used as global coordinates. The radius of the chamber and the longitudinal length of the bellows are a and g, respectively. The minimum and the maximum radial sizes of the bellows are $2R$ and $2R+d$, respectively. Typically, the radial size of the bellows is larger than that of the chamber. Consequently, a gap appears on $(\rho = a, 0<\theta<2\pi, -g/2<z<g/2)$ and forms a space in $(a<\rho, 0<\theta<2\pi, -g/2<z<g/2)$. As shown in the left figure in Fig. 1, let us call it the space cavity.

A scaled figure of the cavity is shown in the right of Fig. 1. The period of the bellows is L. Cartesian coordinates (ξ, η, ζ) are used as local coordinates. Following Ref. [17], the dimensionless variables

$$w = \frac{\eta}{R}, \tag{1}$$

$$u = \frac{2\pi\xi}{L}, \tag{2}$$

$$\varepsilon = \frac{2\pi R}{L} \tag{3}$$

can be introduced and utilized if necessary.

The surface of the bellows is described as

$$w_b = 1 + \Delta(1+\cos\, u) \tag{4}$$

where

$$\Delta = \frac{d}{2R}. \tag{5}$$

2.2. Formal expressions for longitudinal impedance

Let us start with electromagnetic fields derived from the interaction between a beam and the gap. We assume that the beam has a cylindrically uniform density with a radius of σ, and its total charge is 1 C/m. That is, its current density is given by

$$j_z = \beta c(1-\Theta(\rho-\sigma))e^{-jkz+j\omega t}/(\pi\sigma^2) \tag{6}$$

where j is the imaginary unit, $\omega = 2\pi f$, f is the frequency, $\Theta(x)$ is the step function, $k=\omega/\beta c$, $\beta = v/c$, v is the velocity of the beam, and c is the velocity of light.

Now, let us calculate the excited electromagnetic fields. When the radius of chamber a is larger than the gap size g, the fields for $\rho \le a$ are approximately written as [18]

$$E_z = \frac{jcZ_0}{\pi\sigma^2\gamma}\left(\frac{1}{\bar{k}} - \sigma I_0(\bar{k}\rho)K_1(\bar{k}\sigma) - \frac{\sigma I_0(\bar{k}\rho)I_1(\bar{k}\sigma)K_0(\bar{k}a)}{I_0(\bar{k}a)}\right)e^{-jkz} + \frac{V_1}{2\pi}\int_{-\infty}^{\infty} dh\, e^{-jhz}\frac{J_0(\Lambda\rho)}{J_0(\Lambda a)}\frac{2\sin\frac{hg}{2}}{hg} \tag{7}$$

$$H_\theta = \frac{\beta c}{\pi\sigma}\left(K_1(\bar{k}\sigma) + \frac{I_1(\bar{k}\sigma)K_0(\bar{k}a)}{I_0(\bar{k}a)}\right)I_1(\bar{k}\rho)e^{-jkz} + \frac{V_1}{2\pi}\frac{jk\beta}{Z_0}\int_{-\infty}^{\infty} dh\, e^{-jhz}\frac{J_1(\Lambda\rho)}{\Lambda J_0(\Lambda a)}\frac{2\sin\frac{hg}{2}}{hg} \tag{8}$$

$$E_\rho = \frac{cZ_0}{\pi\sigma}\left(K_1(\bar{k}\sigma) + \frac{I_1(\bar{k}\sigma)K_0(\bar{k}a)}{I_0(\bar{k}a)}\right)I_1(\bar{k}\rho)e^{-jkz} + \frac{V_1}{2\pi}j\int_{-\infty}^{\infty} dh\, e^{-jhz}\frac{hJ_1(\Lambda\rho)}{\Lambda J_0(\Lambda a)}\frac{2\sin\frac{hg}{2}}{hg} \tag{9}$$

for $\rho < \sigma$, and

$$E_z = \frac{jcZ_0}{\pi\sigma\gamma}I_1(\bar{k}\sigma)\left(K_0(\bar{k}\rho) - \frac{I_0(\bar{k}\rho)K_0(\bar{k}a)}{I_0(\bar{k}a)}\right)e^{-jkz} + \frac{V_1}{2\pi}\int_{-\infty}^{\infty} dh\, e^{-jhz}\frac{J_0(\Lambda\rho)}{J_0(\Lambda a)}\frac{2\sin\frac{hg}{2}}{hg} \tag{10}$$

$$H_\theta = \frac{\beta c}{\pi\sigma}I_1(\bar{k}\sigma)\left(K_1(\bar{k}\rho) + \frac{K_0(\bar{k}a)I_1(\bar{k}\rho)}{I_0(\bar{k}a)}\right)e^{-jkz} + \frac{V_1}{2\pi}\frac{jk\beta}{Z_0}\int_{-\infty}^{\infty} dh\, e^{-jhz}\frac{J_1(\Lambda\rho)}{\Lambda J_0(\Lambda a)}\frac{2\sin\frac{hg}{2}}{hg} \tag{11}$$

$$E_\rho = \frac{cZ_0}{\pi\sigma}I_1(\bar{k}\sigma)\left(K_1(\bar{k}\rho) + \frac{K_0(\bar{k}a)I_1(\bar{k}\rho)}{I_0(\bar{k}a)}\right)e^{-jkz} + \frac{V_1}{2\pi}j\int_{-\infty}^{\infty} dh\, e^{-jhz}\frac{hJ_1(\Lambda\rho)}{\Lambda J_0(\Lambda a)}\frac{2\sin\frac{hg}{2}}{hg} \tag{12}$$

for $\rho > \sigma$, respectively, where $\bar{k} = k/\gamma$, γ is the Lorentz factor, V_1 is the voltage of the gap, $Z_0(=120\pi)$ is the impedance of free space, $I_m(z)$ and $K_m(z)$ are the modified Bessel functions, $J_m(z)$ is the Bessel function, and $\Lambda = \sqrt{k^2\beta^2 - h^2}$ [19]. The time dependence of the fields is assumed to be harmonic, and it is expressed as the complex exponential $e^{j\omega t}$.

Because the coupling impedance Z_L is defined as the average of the longitudinal electric field (normalized by the beam current) over the beam cross-section, the average value of E_z (expressed by Eq. (7) over ρ) gives the longitudinal impedance as

$$Z_L = Z_{L,sp} + Z_{L,bellows} \tag{13}$$

where $Z_{L,bellows}$ is the impedance of the bellows:

$$Z_{L,bellows} = -\frac{4V_1 I_1(\bar{k}\sigma)\sin\frac{kg}{2}}{c\beta\sigma\bar{k}I_0(\bar{k}a)kg} \tag{14}$$