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Dynamic behavior of thin and thick cracked nanobeams incorporating surface effects

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ABSTRACT

Free transverse vibration of cracked nanobeams is investigated in the presence of the surface effects. Two nanobeam types, thin and thick, are studied using two beam theories, Euler–Bernoulli and Timoshenko. The influences of crack severity and position, surface density, rotary inertia and shear deformation, nanobeam dimension, mode number, satisfying balance condition between the surface layers and the bulk, boundary conditions and satisfying compatibility and boundary conditions with appropriate resultant moment and shear force are studied in details. It is found out that satisfying compatibility and boundary conditions with the resultant moment and shear force in presence of the surface effects and considering surface density neglected in previous work have significant effects on the natural frequencies of cracked nanobeams. In addition, rotary inertia and shear deformation cause a reduction in the crack and surface effects on the natural frequencies.

1. Introduction

According to application of nanobeams as nanosensors, actuators, nanogenerators, transistors, diodes and resonators in nanoelectromechanical systems and in biotechnology [1–5] it is important to investigate the vibrational behavior of nanobeams. In addition, structures at nanometer length scale are known to exhibit size-dependent behavior [6–8]. Since the surface-to-bulk ratio is large in nanostructures accordingly, the surface effects cannot be ignored [9]. Gurtin and Murdoch [10,11] presented a 3D theory based on continuum mechanics concept that takes into consideration the effects of surface energy. In their work, a surface is regarded as a mathematical deformable membrane of zero thickness fully adhered to the underlying bulk material. The equilibrium and constitutive equations for the bulk are the same as those in the classical theory of elasticity. In addition, a set of constitutive equations and the generalized Young–Laplace equation are applied to the surface. Using the model proposed by Gurtin and Murdoch [10,11], He and Lilley [12] investigated the surface effects from surface stress and surface elasticity on the elastic behavior of nanowires in static bending. Lu et al. [13] presented a general thin plate theory including the surface effects which can be used for size-dependent static and dynamic analysis of plate like thin film structures. Nazemnezhad et al. [14] considered the surface effects, including the surface density, the surface stress and the surface elasticity, on the nonlinear free vibration of Euler–Bernoulli nanobeams. In their work, they assumed that the normal stress, σ_{zz}, varies linearly through the nanobeam thickness and satisfies the balance conditions between the nanobeam bulk and its surfaces. It was showed that the effect of the surface density became more by increasing the mode number although it was independent of the vibration amplitude.

In the literatures above, it is assumed that the structures are intact or free from defects, while it is known that defects can change the mechanical behaviors of structures. For example, cracks, as a common defect in structural elements, can reduce the natural frequencies of the structures because they become more flexible in presence of the cracks. Therefore, the understanding and modeling of defects can improve the design of Nanoelectromechanical Systems (NEMS) [15–19]. There are a few studies in which the effects of the defects are considered. In the work done by Luque et al. [15], transverse, atomically sharp surface cracks with circular fronts of different depths were introduced to evaluate their effect on the mechanical strength of the nanowires using molecular dynamic simulation. Longitudinal and flexural vibrations of cracked nanobeams were studied within the framework of the nonlocal Euler–Bernoulli and the nonlocal Timoshenko theories [16–18]. The only work that the transverse vibration of cracked Euler–Bernoulli

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nanobeams was studied in presence of the surface effects was the one done by Hasheminejad et al. [19]. They considered the influences of the surface elasticity and surface tension. Moreover, the balance condition between the surface layers and the nanobeam bulk was not satisfied and the surface density was neglected.

From literature, it is understood that the effects of the rotary inertia and the shear deformation on the free vibrations of cracked nanobeams are not examined when the surface effects are included. Also, the influences of satisfying the balance condition and the surface density effect are not reported. Therefore, in this article, the influences of three parameters, including the rotary inertia and shear deformation, the surface density and satisfying the balance condition, on the free transverse vibration of cracked nanobeams are investigated. To this end, governing equations of the cracked nanobeams incorporating the surface effects are obtained based on the Timoshenko and Euler–Bernoulli theories. A linear variation for the normal stress, σ_{zz}, is assumed in order to satisfy the balance condition. The influences of crack position, crack depth, mode number and dimension of the nanobeam on the natural frequencies of the simply-simply and clamp–clamp nanobeams are examined.

2. Problem formulation

In this section, the governing equations of a nanobeam in presence of the surface effects are derived. To this end, we consider a nanobeam with rectangular cross section with length L $(0 \leqslant x \leqslant L)$, width b $(-0.5b \leqslant y \leqslant 0.5b)$ and thickness, $H = 2h$ $(-h \leqslant z \leqslant +h)$.

2.1. Surface effects

At the micro/nanoscale, the fraction of energy stored in the surfaces becomes comparable with that in the bulk, because of the relatively high ratio of surface area to volume of nanoscale structures; therefore the surface and the induced surface forces cannot be ignored. The constitutive relations of the surface layers, S^+ (upper surface) and S^- (Lower surface), given by Gurtin and Murdoch [10,11] as

$$\begin{aligned}\tau^{\pm}_{\alpha\beta} &= \tau^{\pm}_0\delta_{\alpha\beta} + (\mu^{\pm}_0 - \tau^{\pm}_0)(u^{\pm}_{\alpha,\beta} + u^{\pm}_{\beta,\alpha}) + (\lambda^{\pm}_0 + \tau^{\pm}_0)u^{\pm}_{\gamma,\gamma}\delta_{\alpha\beta} \\ &\quad + \tau^{\pm}_0 u^{\pm}_{\alpha,\beta}, \tau^{\pm}_{\alpha 3} \\ &= \tau^{\pm}_0 u^{\pm}_{3,\alpha}\end{aligned} \quad (1)$$

where $\tau^{\pm}_0$ are residual surface tensions under unconstrained conditions, $\lambda^{\pm}_0$ and $\mu^{\pm}_0$ are the surface Lame constants on the surfaces S^+ and S^- which can be determined from atomistic calculations [20], $\delta_{\alpha\beta}$ is the Kronecker delta and $u^{\pm}_{\alpha}$ are the displacement components of the surfaces S^+ and S^-. If the top and bottom layers have the same material properties, the stress–strain relations of the surface layers, i.e. Eq. (1), can be reduced to the following relation for nanobeams

$$\tau_{xx} = \tau_0 + E^s u_{x,x};\ E^s = 2\mu_0 + \lambda_0;\ \tau_{nx} = \tau_0 u_{n,x} \quad (2)$$

where n denotes the outward unit normal. The equilibrium relations for the surface layers can be expressed in terms of the surface and bulk stress components as

$$\tau_{i\alpha,\alpha} - T_i = \rho_0 \ddot{u}^s_i \quad (3)$$

where $i = x, n, t$; $\alpha = x, t$; ρ_0 denotes surface density; T is the contact tractions on the contact surface between the bulk material and the surface layer; t is the tangent unit vector; and $\ddot{u}^s_i$ denotes the acceleration of surface layers in the i-direction.

In the classical beam theory, the stress component, σ_{zz}, is neglected. However, σ_{zz} must be considered to satisfy the surface equilibrium equations of the Gurtin–Murdoch model. It is assumed that bulk stress, σ_{zz}, varies linearly through the nanobeam thickness as follow

$$\sigma_{zz} = \frac{1}{2}(\sigma^+_{zz} + \sigma^-_{zz}) + \frac{z}{H}(\sigma^+_{zz} - \sigma^-_{zz}) \quad (4)$$

By considering Eq. (2) and satisfying Eq. (3) both of σ^+_{zz} and σ^-_{zz} are obtained. Therefore, Eq. (4) can be rewritten as

$$\begin{aligned}\sigma_{zz} &= \frac{1}{2}(\tau_0 u^+_{z,xx} - \tau_0 u^-_{z,xx} - \rho_0\ddot{u}^+_z + \rho_0\ddot{u}^-_z) \\ &\quad + \frac{z}{H}(\tau_0 u^+_{z,xx} + \tau_0 u^-_{z,xx} - \rho_0\ddot{u}^+_z - \rho_0\ddot{u}^-_z)\end{aligned} \quad (5)$$

2.2. Governing equations of nanobeams

2.2.1. Timoshenko beam theory

The bending moment and vertical force equilibrium equations including rotary inertia, shear deformation and surface effects can be expressed as follow [21]

$$\frac{dM}{dx} + \int_s \tau_{xx,x} z ds - Q = \int_A \rho \ddot{u}_x z dA + \int_s \rho_0 \ddot{u}^s_x z ds \quad (6)$$

$$\frac{dQ}{dx} + \int_s \tau_{nx,x} n_z ds = \int_A \rho \ddot{u}_z dA + \int_s \rho_0 \ddot{u}^s_n n_z ds \quad (7)$$

where τ_{xx} and τ_{nx} are nonzero membrane stresses due to surface energy; ρ is the bulk density. Q and M are the stress resultants defined as

$$Q = \int_A \sigma_{xz} dA;\ M = \int_A \sigma_{xx} z dA \quad (8)$$

Bulk stress–strain relations of the nanobeam can be expressed as

$$\sigma_{xx} = E\varepsilon_{xx} + \nu\sigma_{zz},\ \sigma_{xz} = 2G\varepsilon_{xz} \quad (9)$$

where E is the elastic modulus, ν is the Poisson's ratio and G is the shear modulus. Defining the displacement fields as Timoshenko beam theory

$$u_x = z\phi(x,t),\ u_z = w(x,t) \quad (10)$$

where $\phi(x,t)$ and $w(x,t)$ denote the rotation of cross section and vertical displacement of mid-plane at time t, respectively. So, the nonzero strains are given by

$$\begin{aligned}\varepsilon_{xx} &= \frac{\partial u_x}{\partial x} = z\frac{\partial\phi(x,t)}{\partial x},\ \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \\ &= \frac{1}{2}\left(\phi(x,t) + \frac{\partial w(x,t)}{\partial x}\right)\end{aligned} \quad (11)$$

Substituting Eq. (10) into Eq. (2) we have

$$\tau_{xx} = \tau_0 + zE^s\frac{\partial\phi}{\partial x},\ \tau_{nx} = \tau_0\frac{\partial w}{\partial x}n_z,\ \tau^{\pm}_{nx} = \pm\tau_0\frac{\partial w}{\partial x} \quad (12)$$

The relative bulk stresses can be presented by substituting Eq. (12) into Eq. (5) and then into Eq. (9)

$$\begin{aligned}\sigma_{zz} &= \frac{2z}{H}\left(\tau_0\frac{\partial^2 w}{\partial x^2} - \rho_0\ddot{w}\right),\ \sigma_{xx} \\ &= E\left(z\frac{\partial\phi}{\partial x}\right) + \frac{2\nu z}{H}\left(\tau_0\frac{\partial^2 w}{\partial x^2} - \rho_0\ddot{w}\right),\ \sigma_{xz} = Gk\left(\frac{\partial w}{\partial x} + \phi\right)\end{aligned} \quad (13)$$

where k denotes shear correction coefficient. By substituting Eq. (13) into Eq. (8) and considering Eq. (12), Eqs. (6) and (7) can be rewritten as follow

$$(\rho A + 2b\rho_0)\frac{\partial^2 w}{\partial t^2} = kGA\left(\frac{\partial\phi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + 2b\tau_0\frac{\partial^2 w}{\partial x^2} \quad (14)$$

$$\begin{aligned}&(\rho I + \rho_0 I^*)\frac{\partial^2\phi}{\partial t^2} + kGA\left(\phi + \frac{\partial w}{\partial x}\right) \\ &= (EI + E^s I^*)\frac{\partial^2\phi}{\partial x^2} + \frac{2\nu\tau_0 I}{H}\frac{\partial^3 w}{\partial x^3} - \frac{2\nu\rho_0 I}{H}\frac{\partial^3 w}{\partial x\partial t^2}\end{aligned} \quad (15)$$