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Vortical field amplification and particle acceleration at rippled shocks

F. Fraschetti *,[1]

Departments of Planetary Sciences and Astronomy, University of Arizona, Tucson, AZ 85721, USA

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ABSTRACT

Supernova Remnant (SNR) shocks are believed to accelerate charged particles and to generate strong turbulence in the post-shock flow. From high-energy observations in the past decade, a magnetic field at SNR shocks largely exceeding the shock-compressed interstellar field has been inferred. We outline how such a field amplification results from a small-scale dynamo process downstream of the shock, providing an explicit expression for the turbulence back-reaction to the fluid whirling. The spatial scale of the X-ray rims and the short time-variability can be obtained by using reasonable parameters for the interstellar turbulence. We show qualitatively that such a vortical field saturation might be faster than the acceleration time of the synchrotron emitting energetic electrons.

1. Introduction

The origin of cosmic-rays (CRs) still eludes the theoretical and observational efforts in astroparticle physics since their discovery more than a century ago. Space and ground-based experiments have been providing us with a wealth of multi-wavelength observations to identify the source and investigate the mechanism of acceleration in various energy bands. Individual shell-type Supernova Remnant (SNR) shocks accelerate charged particles and are believed to provide a significant fraction of the power sustaining the observed CR spectrum. Moreover, realistic corrugated shocks travelling in the inhomogeneous interstellar space generate turbulence in the compressed post-shock fluid.

The inhomogeneity of the unshocked ISM observed over several scales [2] is expected to deform the shock surface rippling the initial local planarity up to scales many orders of magnitude greater than the thermal ion inertial length. *HST* observations of SN1006 [25] constrain the length-scale of the shock ripples to $10^{16}-10^{17}$ cm. We focus on the interaction of a non-relativistic SNR rippled shock with the turbulence upstream of the shock, disregarding the contribution of accelerated particles at the shock, as justified later.

From detection of non-thermal X-ray rims [29,4], rapid time-scale variability of X-ray hot spots [28] and γ-ray emission in extended regions [1], a magnetic field at the shock far exceeding the theoretically predicted shock-compressed field has been inferred. Whether or not such a magnetic field amplification in SNR is to be associated with energetic particles at the shock is still subject of controversy.

Magnetic field amplification might be also relevant to *in situ* measurements of the plasma downstream of the solar-wind termination shock [8], where fluctuations have been measured of the same order as the mean, or to radio observations of Mpc scale shocks at the edge of galaxy clusters [7]. Strong magnetic fields are also required in Gamma-Ray Bursts (GRB) and Active Galactic Nuclei (AGN) outflows to enable sufficient production of non-thermal radiation. In the ISM magnetic energy density and thermal pressure are typically comparable and both amount to a fraction $10^{-9}-10^{-7}$ of the total internal energy density (including rest mass). Therefore, a compression by an ultra-relativistic shock (bulk Lorentz factor ~ 100) cannot produce the fraction $10^{-3}-10^{-1}$ predicted by GRB phenomenological models of afterglow light curves [23].

The passage of an oblique non-relativistic shock through inhomogeneous medium has been known for longtime to generate vorticity in the downstream flow [17]; in a conducting fluid the turbulent motion at scale l with fluid velocity v_l and local density ρ leads exponentially fast to an amplified magnetic field $B^2 = 4\pi\rho v_l^2$ [21]. The encounter of a shock surface with a density clump, also called Richtmyer–Meshkov (RM) instability [6], has been also extensively investigated in plasma laboratory experiments (see Ref. [11] and references therein).

Recent numerical 2D-MHD simulations have shown that such an amplification can be very efficient [15,16]. Ideal MHD applied to 2D rippled shocks has shown that the ISM turbulence might amplify exponentially fast the upstream magnetic field with a growth rate depending on shock and upstream medium properties [12]. Such an amplification is expected to occur downstream of the blast wave, regardless of the presence of shock-accelerated particles. Magnetic field may also be enhanced by field line stretching due to

* Correspondence address: Department of Planetary Sciences, University of Arizona, Tucson, AZ 85721, USA. Tel.: +1 520 621 4534.
E-mail address: ffrasche@lpl.arizona.edu
[1] Also, Associated Member to LUTh, Observatoire de Paris, CNRS-UMR8102 and Université Paris VII, F-92195, France.

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Rayleigh–Taylor (RT) instability [18] at the interface between the ejecta and the interstellar medium, i.e., far downstream of the shock. In contrast to the vortical turbulence, late-time RT turbulence might be affected by the highest energy particle gyrating in the downstream fluid far from the shock [14]. However, RT structures are unlikely to reach out the blast wave (Ref. [14] and references therein) and therefore to interact with vortical turbulence. Thus the dynamo amplification occurring locally behind the shock can be temporally and spatially disentangled from the field line stretching due to RT instability.

Two-dimensional simulations of relativistic shocks [22] show that small-scale dynamo can operate also downstream of the shocks with bulk Lorentz factor of a few unities. This suggests that the dynamo action downstream of shocks might shed light on the energy equipartition at magnetized shocks of AGN and Gamma-Ray Bursts.

2. Macroscopic approach to rippled shock

Constitutive equations: We consider the propagation of a 2D non-relativistic shock front in an inhomogeneous medium. Within the ideal MHD approximation, i.e., with no viscosity or heat conduction, the time evolution of the fluid velocity $\mathbf{v} = \mathbf{v}(x, y, t)$ and the magnetic field $\mathbf{B} = \mathbf{B}(x, y, t)$ is given, for infinitely conductive fluid, by

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{\nabla P}{\rho} + \frac{1}{4\pi\rho}[\mathbf{B} \times (\nabla \times \mathbf{B})] = 0 \\ \partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) \end{cases} \quad (1)$$

where ρ, P are respectively density and hydrodynamic pressure of the fluid (here $\partial_t = \partial/\partial t$). Note that the current density carried by CRs is here neglected: we aim to identify the growth of the magnetic energy as generated by the vortical motion of the background fluid only. Plasma heating by the shock might reduce the energy deposited in the magnetic turbulence and will be considered in a forthcoming publication.

Vorticity downstream of MHD shock: The vorticity shock-generated is transported along the flow "frozen" into the fluid in the inviscid approximation (Helmholtz–Kelvin theorem). The medium upstream of the shock has $\omega = 0$. The vorticity is calculated in the region downstream of the shock at a distance from the shock large enough such that the shock is infinitely thin, i.e., the thickness of the shock is much smaller than the local curvature radius at every point of the shock surface [12].

At a rippled shock the MHD Rankine–Hugoniot jump conditions cannot be applied globally as the directions normal and tangential vary along the shock surface. For a 2D shock propagating at average in the direction x (all quantities are independent of z, see Fig. 1), from the velocity field of the flow $\mathbf{v} = (v_x, v_y, 0)$, the vorticity is given by $|\omega| = |\nabla \times \mathbf{v}| = \omega_z$. We use a local natural coordinate system $(\hat{n}, \hat{s})$, where $\hat{n} = (\cos\vartheta(t,s), \sin\vartheta(t,s))$ is the coordinate along the normal to the shock surface, $\hat{s} = (\sin\vartheta(t,s), -\cos\vartheta(t,s))$ is the coordinate parallel to the shock surface (Fig. 1). We consider a seed-magnetic field upstream uniform and normal to the average direction of motion ($\mathbf{B}_0 = (0, B_0^y, 0)$, or $B_n = B_0 \sin\vartheta$ and $B_s = -B_0 \cos\vartheta$, see Fig. 1).

The turbulent field is assumed to be much greater than the shock-compressed field in the downstream flow, in agreement with observations, so that the amplification is efficient at the smallest scales (see Section 3). Thus, the vorticity produced downstream of a 2D shock propagating in an inhomogeneous medium with a uniform perpendicular upstream magnetic field (same as for parallel shock [12]) can be recast, neglecting obliqueness, in a simple form (we use $\partial_{x_i} = \partial/\partial x_i$):

$$|\delta\omega_z| = \frac{r-1}{r}\left[\left(\frac{C_r}{\rho}\right)_u \partial_s \rho + \partial_s C_r\right] - \frac{B_n \delta B_s}{4\pi\rho C_r}\partial_s \vartheta, \quad (2)$$

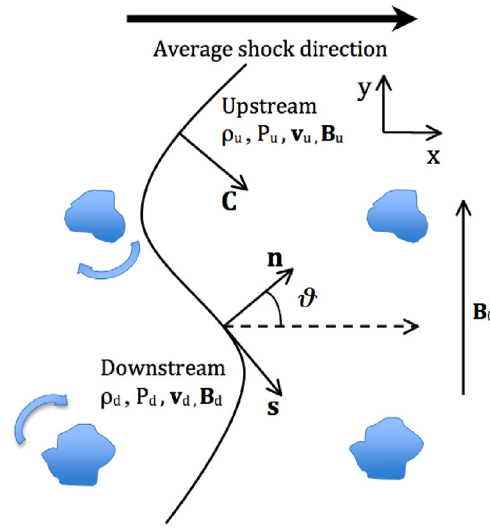

Fig. 1. Encounter of a shock surface with density enhancement regions: forward and lagging behind regions are formed that generate vorticity in the downstream fluid.

where $r = \rho_d/\rho_u$ is the density compression ratio at the shock, C_r is the shock speed relative to the upstream frame, δB_s is the jump across the shock of the magnetic field in the direction locally tangential to the shock surface including the Rankine–Hugoniot compressed seed field and the turbulently amplified field and B_n is the component in the direction locally normal to the shock surface including the unchanged Rankine–Hugoniot and the turbulent components.

Turbulent field amplification: The vortical turbulence described in the previous sub-section exponentially amplifies the total magnetic field. Since the amplification time-scale is of the order of the smallest eddies turnover time [3], the saturation occurs much faster at small-scale [19]. This is the key feature of the small-scale dynamo. The unperturbed field is initially too weak to affect the fluid velocity field and the turbulent field grows exponentially fast, until the magnetic energy produces non-negligible effects on the velocity field and then saturates.

The small-scale dynamo theory predicts that the turbulent field obeys an unbounded exponential amplification at a rate β [19,20]: $d\varepsilon/dt = 2\beta\varepsilon$, where $\varepsilon = B^2/8\pi\rho$ is the total magnetic energy per unit of mass, including seed and turbulent fields. As shown in Ref. [19], the isotropy and homogeneity of the fluid velocity correlation entails the following simple relation between the amplification rate of ε and the vorticity generated downstream of the shock: $\beta \simeq (\pi/3)\delta\omega_z$ (Fig. 2).

If we recast Eq. (2) as $|\delta\omega_z| = (3/\pi)(\tau^{-1} - \alpha\varepsilon)$, then ε satisfies

$$\frac{d\varepsilon}{dt} = 2(\tau^{-1} - \alpha\varepsilon)\varepsilon \quad (3)$$

where $\tau^{-1} = (\pi/3)(r - 1/r)[(C_r/\rho)_u \partial_s \rho + \partial_s C_r]$ is the local growth rate of ε and $\alpha = (2\pi/3)\partial_s \vartheta / C_r$ is the local back-reaction; the initial condition for Eq. (3) is $\varepsilon(0) = \varepsilon_0 = v_A^2/2 = B_0^2/8\pi\rho$. In Eq. (3) we have assumed that the turbulence dominates over B_0, i.e., $\delta B_s/\sqrt{8\pi\rho} \sim \sqrt{\varepsilon}$ and $B_n/\sqrt{8\pi\rho} \sim \sqrt{\varepsilon}$: the turbulence grows isotropically downstream at the shock curvature scale as a consequence of the isotropy of the flow velocity field [19].

Neglecting the time dependence of τ (the magnetic modes grow slowly for initially weak field [19]), the solution is readily found [12]

$$\frac{\varepsilon}{\varepsilon_0}(t) = \left(\frac{B}{B_0}\right)^2(t) = \frac{e^{2t/\tau}}{1 - \alpha\tau(1 - e^{2t/\tau})v_A^2/2}, \quad (4)$$

for a uniform average interstellar matter density.