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Refined equivalent single layer formulations and finite elements for smart laminates free vibrations

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ABSTRACT

A family of 2D refined equivalent single layer models for multilayered and functionally graded smart magneto–electro-elastic plates is presented. They are based on variable kinematics and quasi-static behavior for the electromagnetic fields. First, the electromagnetic state of the plate is determined by solving the strong form of the electromagnetic governing equations coupled with the corresponding interface continuity conditions and external boundary conditions. The electromagnetic state is then condensed into the plate kinematics, whose governing equations can be written using the generalized principle of virtual displacements. The procedure identifies an effective elastic plate kinematically equivalent to the original smart plate. The effective plate is characterized by inertia, stiffness and loading properties which take the multifield coupling effects into account through their definitions, which involve the electromagnetic coefficients appearing in the smart materials constitutive law. The proposed model extends the techniques and tools available for the assessment of the mechanical behavior of multilayered composite plates to smart laminates. Additionally, finite elements for the proposed single layer models are formulated and validated against available benchmark 3D solutions.

1. Introduction

Multilayered (ML) composite structures are widely employed in many engineering fields like aerospace, automotive, biomedical, and robotics, in force of their excellent structural properties and versatility. Generally, they are constituted by layers of different orthotropic materials and their configuration leads to complex stress states and failure mechanisms, whose assessment is crucial for the their successful design. The evolution of the multilayered structures concept led to functionally graded (FG) structures, which present graded spatial changes in their material properties. This kind of materials have some advantages with respect to laminated composites, essentially consisting in the absence of the abrupt mismatches at the layers interfaces that may give raise to large interlaminar stresses responsible of damage initiation and propagation. The analysis of ML and FG graded structures is a difficult task and different approaches have been proposed in the literature, especially for plate structures which are the most employed in applications. In particular two-dimensional modeling of ML and FG plates has received much attention and different order refined layer-wise or equivalent single layer theories have been proposed and solved by analytical and/or numerical techniques [1,2].

In recent years, the developments of *smart materials* open towards the possibility to provide structures with multi-functional capabilities, related to the inherent coupling between mechanical and electrical fields, like in piezoelectrics, or among mechanical, electrical and magnetic fields, like in magneto–electro-elastic (MEE) materials. The possibility of coupling the different physical fields can be exploited in transducer applications, structural health monitoring, vibration control, energy harvesting and other applications. In this framework, multilayered and/or functionally graded structures may even be more effective than bulk materials. Thus, reliable and efficient modeling tools are required for the analysis and design of these smart plates, especially for free vibrations and dynamics which are common in many applications.

Modeling of smart piezoelectric composite laminates received much attention as reported in the comprehensive reviews by Gopinathan et al. [3] and Kapuria et al. [4]. More recently MEE materials are attracting increasing consideration from academic and industrial audiences [5]. Focusing on magneto–electro-elastic plates free vibrations problem, exact 3D solutions have been proposed for both ML [6] and FG [7,8] plates. The state-vector approach [9–12], the Pagano method [13] and asymptotic approach [14] have been also used. It is worth noting that the approaches developed

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for multilayered plates are usually extended to the analysis of FG plates by incorporating the properties through-the-thickness variation into the original formulation (e.g. Ref. [8]) or, often, using the concept of approximate laminate model where the FG plate is artificially divided into individual small thickness layers having constant material properties determined by suitable averages over the thickness (e.g. Refs. [9,7,15]). The mentioned solutions generally refer to rectangular geometry and simply-supported boundary conditions, providing very useful benchmarks. For more general configurations numerical solutions are needed and the finite element method appears an effective choice. Fully-coupled 3D finite element solutions for smart multilayered plates have very high computational costs and therefore 2D efficient plate theories and the corresponding finite element solutions have been developed to reduce the analysis effort, while preserving a suitable level of accuracy.

In the framework of 2D multilayered smart plates theories, finite elements solutions based on layer-wise modeling (LW) have been used [16,17]. The layer-wise approach surely enables high accuracy but the associated computational cost grows as the number of layers increases. Additionally, layer-wise based finite elements are somehow difficult to be implemented into finite element commercial codes. On the other hand, equivalent single-layer (ESL) plate theories do not present these drawbacks as their solution complexity is independent from the number of layers and the effort required for their numerical solution is therefore more affordable; however they are generally less accurate than the layer-wise ones, especially for thick laminates whose analysis demand refined higher order theories and when transverse fields components are involved. To formulate and implement refined higher order theories for multilayered plates, Carrera proposed a powerful approach known as CUF (Carrera Unified Formulation) whose underlying ideas, principles and implementation issues for mechanical problems can be found in Refs. [18,19]. CUF offers a systematic procedure to generate different order 2D refined plate models and the corresponding finite elements, considering the order of the theory as a free parameter of the formulation. It was extensively applied to piezoelectric plates [20–26] and, finally, the technique was also extended to magneto–electro-elastic problems [27–29].

Finite element solutions for smart plates based on ESL theories have been generally formulated taking the electric and magnetic potential as independent state variable of the problem (e.g. [29,30]). More recently, Milazzo [31] proposed modeling of magneto–electro-elastic plates through an *effective mechanical plate* resulting from the condensation of the electromagnetic state into the mechanical variables and developed the corresponding finite element based on the FSDT [32,33].

The development of ESL effective mechanical plate models for smart multilayered and functionally graded plates results appealing as such a modeling strategy could take advantage of the well-established solution techniques available for the mechanics of multilayered plates with advantageous computational costs with respect to more sophisticated layer-wise approaches and could provide useful tools in the design and optimization of smart structures.

Basing on this rationale, the objective of the present work is to derive a family of ESL plate models for smart magneto–electro-elastic laminates based on refined higher order plate theories. In particular, generalizing the approach presented in previous author's papers [31,34] for thin to moderately thick laminates, the smart plate is modeled through an effective ESL mechanical plate obtained by preliminarily condensing the electromagnetic state to the plate kinematics. Models with variable kinematics are derived by using the technique proposed by Carrera and Demasi [19], which allows to systematically built different order refined plate theories. These are then solved by introducing the corresponding finite element formulation. The development of different order refined theories with the associated finite elements extends the applicability of the proposed equivalent single-layer modeling for advanced magneto–electro-elastic smart laminates to general geometrical configurations (in-plane geometry and thickness) and loading and boundary conditions.

The paper is organized as follows. The basic assumptions and governing equations are introduced in Section 2. Then, the ESL theory for smart layered plates with variable kinematics is formulated in Section 3 and the corresponding finite elements are developed in Section 4. In Section 5, validation of the finite elements solutions for free vibrations of multilayered and functionally graded plates is presented.

2. Governing equations and basic assumptions

Consider a multilayered plate referred to a coordinate system with the x_1 and x_2 coordinates spanning its mid-plane Ω, whose boundary is denoted by $\partial\Omega$. The x_3 axis is directed along the thickness. The plate consists of N_L layers of homogeneous and orthotropic magneto–electro-elastic materials having poling direction parallel to the x_3-axis and principal directions rotated by an angle θ with respect to the x_1- and x_2-axes. The cases of piezoelectric and purely elastic layers are covered as subcases of the more general magneto–electro-elastic behavior. The plate has thickness h and the kth layer of the laminate has constant thickness t_k, with $x_3 = h_{k-1}$ and $x_3 = h_k$ representing the quote of its bottom and top faces, respectively. The bottom and top surfaces of the plate are identified by the coordinate $x_3 = h_l$ and $x_3 = h_u$, respectively. See Fig. 1 for the geometrical scheme of the plate. The plate is subjected to mechanical loads and to electric and magnetic conditions acting on its top and bottom surfaces.

2.1. Primary variables and gradient equations

As the elastic waves propagate several order of magnitude slower than the electromagnetic ones, the quasi-static approximation for the electromagnetic fields can be considered. Therefore, to describe the plate response the displacements $\boldsymbol{u} = \{u_1 \quad u_2 \quad u_3\}^T$, the electric potential Φ and the magnetic scalar potential Ψ are used as primary variables.

The strain field $\boldsymbol{\varepsilon}$ is partitioned into the in-plane components $\boldsymbol{\varepsilon}_p = \{\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{12}\}^T$ and out-of-plane components $\boldsymbol{\varepsilon}_n = \{\varepsilon_{23} \quad \varepsilon_{13} \quad \varepsilon_{33}\}^T$; accordingly the linear strain–displacement relationships read as

$$\boldsymbol{\varepsilon}_p = \mathcal{D}_p \boldsymbol{u} \tag{1a}$$

$$\boldsymbol{\varepsilon}_n = \mathcal{D}_n \boldsymbol{u} + \mathcal{D}_{x_3} \boldsymbol{u} \tag{1b}$$

where the following differential operators are introduced

$$\mathcal{D}_p = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_2} & 0 \\ \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} & 0 \end{bmatrix} \tag{2a}$$

$$\mathcal{D}_n = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x_2} \\ 0 & 0 & \frac{\partial}{\partial x_1} \\ 0 & 0 & 0 \end{bmatrix} \tag{2b}$$

$$\mathcal{D}_{x_3} = \begin{bmatrix} \frac{\partial}{\partial x_3} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_3} & 0 \\ 0 & 0 & \frac{\partial}{\partial x_3} \end{bmatrix} = \mathcal{D}_i \frac{\partial}{\partial x_3} \tag{2c}$$

being $\mathcal{D}_i$ the 3×3 identity matrix.