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The combined influences of heterogeneity and elastic foundations on the nonlinear vibration of orthotropic truncated conical shells

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ABSTRACT

The aim of the present paper is to study the nonlinear vibration of heterogeneous orthotropic truncated conical shells resting on the Winkler–Pasternak elastic foundations. The formulation is based on the Donnell shell theory, exponential-law distribution of orthotropic material properties and von Karman geometric nonlinearity. The basic equations are reduced to a time dependent geometrical nonlinear differential equation and solved using homotopy perturbation method (HPM). Finally, the influences of elastic foundations, heterogeneity, material orthotropy and shell characteristics on the nonlinear vibration of the truncated conical shell are studied.

1. Introduction

Heterogeneous materials are widely used in engineering design and modern technology to increase the strength of the construction. The heterogeneity of the materials may stem from effects of the humidity, manufacturing process, production techniques, surface and thermal polishing processes, operating under radiation, etc. Various models for heterogeneity of shell materials have been proposed in the literature and a detailed discussion is given in Refs. [1,2]. The high strength composites fabricated by mixing two or more materials, such as glass–epoxy or boron-epoxy used in steel alloys for making control rods in nuclear reactors, are recent innovations. The mechanical properties of such materials may vary depending on the spatial coordinates, either continuously or intermittently, in an arbitrary specified way [3,4]. Moreover, some phases of the heterogeneous materials may have orthotropic characters. For example, in graded composite materials, graded regions are treated as series of perfectly bonded composite layers, each layer being assigned slightly different properties. Continuously varying the volume fraction of fibers in the transverse direction may also lead to the orthotropic functionally graded or heterogeneous orthotropic materials [4]. In order to take the oriented structure of FGMs into account in solid mechanics analyses, these materials are generally modeled as orthotropic with principal directions. In recent years, various methods have been developed for the study of solutions of bending, vibration, stability and thermo-elastic problems of FG orthotropic composite structures. Wu and Lee [5] studied differential quadrature solution for the free vibration analysis of laminated conical shells with variable stiffness. Pan [6] presented an exact solution for a simply supported rectangular FG anisotropic laminated plate using the pseudo-Stroh formalism extending Pagano's solution to the FG plates. Chen et al. [7] studied the free vibration of simply supported, fluid-filled cylindrically orthotropic FG cylindrical shells with arbitrary thickness based on the three-dimensional equations of elasticity. Batra and Jin [8] studied natural frequencies of a FG graphite/epoxy rectangular plate based on first order shear deformation. Pelletier and Vel [9] investigated an exact solution for the steady-state thermo-elastic response of FG orthotropic cylindrical shells using Flügge and Donnell shell theories. Ramirez et al. [10] examined static analysis of FG orthotropic plates using a discrete layer approach in combination with the Ritz method. Ootao and Tanigawa [11] presented three-dimensional solution for transient thermal stresses of an orthotropic FG rectangular plate using Laplace and finite cosine transformation methods. Baron [12] investigated propagation of elastic waves in the anisotropic hollow cylinder with elastic properties (stiffness coefficients and mass density) functionally varying in the radial direction based on the sextic Stroh's formalism and an analytical solution, the matricant, explicitly expressed under the Peano series expansion form. Peng and Li [13] studied the influence of orthotropy and gradient on the elastic field in particular the hoop stress distribution in hollow annular plates rotating at constant angular speed about its axis. Malekzadeh and Heydarpour [14] examined free vibration analysis of rotating functionally graded truncated conical shells with FSDT using the

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generalized differential quadrature (GDQ) method Sofiyev [15,16] examined influences of heterogeneity and orthotropy on the frequency–amplitude characteristics of single-layer and cross-ply laminated truncated conical shells, respectively, using the semi-inverse method. Almost in all analyses performed for the orthotropic FG materials and exponential laws were used.

A plentiful number of shells resting on or surrounded by elastic foundations are important in structural engineering and have wide applications in other fields of technology. Such shell structures may be found in various kinds of industrial applications such as storage tanks, pressure vessels, water pipes, pipe lines and casing pipes, processing equipments, tunnels, and deep sea pressure vessels. One-and two-parameter models for the soil underneath the shell are introduced to model the foundation. The Pasternak model or the two-parameter model is widely adopted to describe the mechanical behavior of foundations, and the well known Winkler model is one of its special cases [17–19]. Mathematically, both formulation and solution of vibration problems of shells are more complicated (use of theory of elasticity and theory of shell, formulation of medium-shell interface conditions, development of solution methods, etc.). The researches are limited on the vibration of homogeneous orthotropic shells resting on the Winkler–Pasternak type elastic foundations than homogeneous isotropic shells. Paliwal and Bhalla [20] presented large amplitude free vibration analysis of non-circular homogeneous isotropic cylindrical shells on the Pasternak type foundation. Paliwal et al. [21] studied an orthotropic shallow cylindrical shell lying on a Pasternak type elastic foundation and the influences of shell geometry, foundation parameters and edge conditions on Load/deflection curves are discussed. Paliwal and Pandey [22] investigated the influences of Pasternak foundation parameters, orthotropy of materials represented by elastic moduli ratio, axial wave parameter, and circumferential wave number on three eigen-frequencies. Nie [23] studied analysis of nonlinear behavior of imperfect shallow spherical shells on the Pasternak type elastic foundation by the asymptotic iteration method. Civalek [24] examined geometrically nonlinear dynamic analysis of doubly curved isotropic shells resting on elastic foundation by a combination of harmonic differential quadrature-finite difference methods. The work of Wang and Ni [25] includes free vibrations of fluid-conveying pipes on a Pasternak type elastic foundation in the large-amplitude using the principle of conservation of total energy. Kurpa et al. [26] proposed a method to solve geometrically nonlinear bending problems for thin orthotropic shallow shells and plates interacting with the Winkler–Pasternak type foundations under transverse loading based on the Ritz and R-function methods. Tornabene [27] studied free vibrations of anisotropic doubly-curved shells and panels of revolution with a free-form meridian resting on Winkler–Pasternak elastic foundations using the first-order shear deformation theory (FSDT), Reissner–Mindlin and Toorani–Lakis theory and generalized differential quadrature (GDQ) method. Bakhtiari-Nejad and Bideleh [28] solved the nonlinear free vibration problem of pre-stressed circular cylindrical shells on the Winkler–Pasternak type foundations using the nonlinear Sanders–Koiter and Love shell theories, and Rayleigh–Ritz procedure. Tornabene et al. [29] investigated effect of Winkler–Pasternak type foundations on the static and dynamic analyses of laminated doubly-curved and degenerate shells and panels with FSDT using the generalized differential quadrature (GDQ) method.

In the last few years, FG shells resting on or surrounded by elastic foundations, have begun to use in a variety of the engineering structures, such as petrochemical and marine industries as well as mechanical, nuclear and civil engineering applications. The influence of elastic foundations on the linear and nonlinear vibrations of FG isotropic shells is sufficiently well studied in the literature. Sheng and Wang [30] studied the thermal vibration, buckling and dynamic stability of functionally graded cylindrical shells embedded in an elastic medium based on the FSDT considering rotary inertia and transverse shear strains. Bagherizadeh et al. [31] presented the mechanical buckling of functionally graded material cylindrical shell that is embedded in an outer elastic medium and subjected to combined axial and radial compressive loads using higher-order shear deformation shell theory considering the transverse shear strains. Shen [32] investigated the large amplitude vibration behavior of a shear deformable FG cylindrical shell of finite length embedded in a large outer elastic medium and in thermal environments. Sofiyev and his co-authors [33,34] studied the linear and nonlinear vibration of FG isotropic truncated conical shells resting on the Winkler–Pasternak elastic foundations. Duc [35] investigated nonlinear dynamic response of imperfect eccentrically stiffened FG double curved shallow shells on an elastic foundation. Duc and Quan [36] presented nonlinear dynamic analysis of imperfect FG double curved thin shallow shells with temperature-dependent properties on an elastic foundation. Dung et al. [37] presented an analytical approach to investigate the mechanical buckling load of eccentrically stiffened functionally graded truncated conical shells surrounded by elastic medium and subjected to axial compressive load and external uniform pressure.

However, researches for heterogeneous orthotropic structures such as functionally graded orthotropic plates and shells resting on elastic foundations are rare in the literature. Morimoto and Tanigawa [38] studied the stability of infinite inhomogeneous orthotropic plates on an elastic foundation under in-plane compression. Lal [39] investigated the vibration of non-homogeneous orthotropic rectangular plates of varying thickness resting on Pasternak elastic foundation. Sofiyev et al. [40] presented the effect of the two-parameter elastic foundation on the linear critical loads and frequency parameters of non-homogeneous orthotropic shells. The work of Najafov et al. [41] includes the linear torsional vibration and stability of functionally graded orthotropic cylindrical shells on elastic foundations. Shariyat and Asemi [42] presented three-dimensional nonlinear elasticity-based 3D cubic B-spline finite element shear buckling analysis of rectangular orthotropic FG plates surrounded by elastic foundations.

Until recently, analytical techniques for solving nonlinear vibration problems have been dominated by the perturbation methods, which have found wide applications in different fields of engineering [43–47]. But, like other nonlinear analytical techniques, traditional perturbation methods have their own limitations (all perturbation methods based on small parameters; the determination of small parameters requires special techniques; in most cases, perturbation methods are valid only for small values of the parameters). It is obvious that all these limitations come from the small parameter assumption. So it is very necessary to develop a kind of new nonlinear analytical method which does not require small parameters at all. The homotopy technique, or the continuous mapping technique, embeds a parameter p that typically ranges from zero to one. When the embedding parameter is zero, the equation is one of a linear system, when it is one, the equation is the same as the original one. So the embedded parameter $p \in [0, 1]$ can be considered as a small parameter. The coupling method of the homotopy technique and the perturbation technique is called the homotopy perturbation method (HPM) [48,49]. The basic knowledge on the homotopy perturbation method has been presented in the studies of He [50,51]. The present method gives exactly same results as the traditional perturbation methods and can take the full advantage of them He [50]. There are some attempts that focus on solving the partial differential