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Large-amplitude vibration of non-homogeneous orthotropic composite truncated conical shell

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ABSTRACT

In this study, the large-amplitude vibration of non-homogenous orthotropic composite truncated conical shell is investigated. It is assumed that the Young's moduli and density of orthotropic materials vary exponentially through the thickness direction. The basic equations of non-homogenous orthotropic truncated conical shell are derived using the finite deflection theory with von Karman–Donnell-type of kinematic non-linearity. Then, foregoing equations are solved using the Superposition principle, Galerkin and Semi-inverse methods and the frequency- amplitude relationship is found. Finally, carrying out some computations, the effects of non-homogeneity, orthotropy and conical shell characteristics on the nonlinear vibration characteristics have been studied.

1. Introduction

In recent years, new types of composite materials have been used in engineering and many investigations consider non-homogeneous orthotropic materials. In various technological situations are demanding that the non-homogeneity of orthotropic materials should be taken into account for the vibration behavior of structural elements. The non-homogeneity of the materials stems from the effects of humidity, surface and thermal polishing processes and methods of production, which render the physical properties of materials, vary from point to point (random, piecewise continuous or continuous functions of coordinates). Furthermore, certain parts of structural elements have to operate under radiation and elevated temperatures and which cause non-homogeneity in the material, i.e., the constants of the material become functions of space variables. When non-homogeneous materials deform, they retain their shapes up to the point of rupture. Hence, in the computations of structural members made of such materials, the fundamental relations and governing equations of deformable body mechanics are applicable [1–3]. Up till now, several studies have been devoted to the vibration behavior of composite orthotropic structures with variable material properties and reported in references [4–13], to mention a few. In these references various model such as linear, quadratic and exponential for the Young moduli and density of the plate and shell materials have been considered. Above mentioned studies are based on the small deflection theory.

Extensive use has been made of conical shells in various practical applications particularly in aerospace, marine and structural engineering. The vibratory characteristics of conical shells such as aircraft structures and turbo machinery blades are critical for the performance and safety of these structures. Therefore, numerous studies dealing with linear free vibration analysis of homogeneous isotropic [14–20] and orthotropic [21–23] truncated conical shells using thin-shell theory appear in the open literature. In some applications, the vibration response of composite shells calculated by linear theory is inaccurate. Thus, when the vibration amplitude becomes comparable to the shell thickness, a nonlinear theory should be used. There are some important studies on the vibration analysis of perfect and imperfect homogeneous composite shells in the large deformation [24–38]. A complete survey on this subject can be found in Refs. [39–43]. As the geometrical non-linearity is taken into account in the motion equations of non-homogenous shells, unpredictable behaviors may be occur. A review of the literature shows that few studies have been carried out to investigate the vibration of non-homogeneous composite shells in the large deflection [44,45].

Because of the combined effects of orthotropy and non-homogeneity, it is extremely difficult to obtain solutions for non-linear vibration problems of non-homogenous materials with anisotropic properties. From the literature survey, one can see that the non-linear vibration problem of non-homogeneous orthotropic truncated conical shell have not been investigated previously. Therefore, it is very important to develop an accurate, reliable analysis towards the understanding of the non-linear vibration characteristics of the non-homogenous composite structures. In this paper, the

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large-amplitude vibration behaviors of non-homogeneous orthotropic conical shells are investigated by using Donnell shell theory and the non-linear strain–displacement relations of the finite deflection. The frequency–amplitude relationship for the non-homogeneous orthotropic truncated conical shell is obtained using the Superposition principle, Galerkin and semi-inverse methods. Numerical results show various effects of the non-homogeneity, orthotropy, non-homogeneous compositional profiles and conical shell characteristics on the dimensionless non-linear frequency parameter or frequency–amplitude characteristics.

2. Governing equations

A truncated conical shell with thickness, h, and semi-vertex angle, γ, is made of the non-homogeneous orthotropic material. The curvilinear coordinate system is defined as ($S\theta z$), where S and θ coincides with generator and circumferential directions, respectively, and z is perpendicular to $S-\theta$ plane and its direction is inwards normal of the conical shell, as shown in Fig. 1. Here R_1 and R_2 indicate the radii of the cone at its small and large ends, respectively. S_1 and S_2 are the distances from the vertex to the small and large bases, respectively. The axes of orthotropy are parallel to the curvilinear coordinates S and θ. Ψ be the stress function for the stress resultants defined by $N_S=\Psi_{,\theta_1\theta_1}/S^2+\Psi_{,S}/S, N_\theta=\Psi_{,SS}, N_S=-\Psi_{,S\theta_1}/S+\Psi_{,\theta_1}/S^2$, where $\theta_1=\theta sin\gamma$ and a comma denotes partial differentiation with respect to the corresponding coordinates.

The non-homogeneity of the material of the conical shell is assumed to arise due to the variation of Young's moduli, shear modulus and density along the thickness direction z as [1,9,11] $[E_1(Z),E_2(Z),G(Z)]=\bar{\varphi}_1(Z)[E_{01},E_{02},G_0]$; $\rho(Z)=\bar{\varphi}_2(Z)\rho_0$; $Z=z/h$, where E_{01} and E_{02} are the Young's moduli in S and θ directions, respectively, G_0 is the shear modulus and ρ_0 is density of the homogeneous orthotropic materials. Additionally, $\bar{\varphi}_i(Z)=1+\mu\varphi_i(Z),\ (i=1,2)$, where $\varphi_i(Z)$, (i = 1,2) are the continuous functions of the non-homogeneity defining the variation of the Young's moduli, shear modulus (i = 1) and density (i = 2), satisfying the conditions $|\varphi_i(Z)|\leqslant 1 (i=1,2)$, and μ is a non-homogeneity coefficient, satisfying $0\leqslant\mu\leqslant 1$.

In this study, the non-homogeneity function of the orthotropic material of the truncated conical shell is assumed to be exponential function which, $\varphi_i(Z)=e^{-0.1|Z|}\cos(vZ)$, ($i$ = 1, 2), where v is the non-homogeneity parameter [9].

By using large deflection shell theory, the non-linear motion and strain compatibility equations of non-homogeneous truncated conical shells are given as follows [40,45]:

$$\frac{\partial^2 M_S}{\partial S^2}+\frac{2}{S}\frac{\partial M_S}{\partial S}+\frac{2}{S}\frac{\partial^2 M_{S\theta}}{\partial S\partial\theta_1}-\frac{1}{S}\frac{\partial M_\theta}{\partial S}+\frac{2}{S^2}\frac{\partial M_{S\theta}}{\partial\theta_1}+\frac{1}{S^2}\frac{\partial^2 M_\theta}{\partial\theta_1^2}+\frac{\cot\gamma}{S}N_\theta +N_S\frac{\partial^2 w}{\partial S^2}-\frac{N_\theta}{S}\left(\frac{1}{S}\frac{\partial^2 w}{\partial\theta_1^2}+\frac{\partial w}{\partial S}\right)-2N_{S\theta}\left(\frac{1}{S}\frac{\partial^2 w}{\partial S\partial\theta_1}-\frac{1}{S^2}\frac{\partial w}{\partial\theta_1}\right) =\rho_1 h\frac{\partial^2 w}{\partial t^2} \quad (1)$$

$$\frac{\cot\gamma}{S}\frac{\partial^2 w}{\partial S^2}-\frac{2}{S}\frac{\partial^2 e_{S\theta}}{\partial S\partial\theta_1}-\frac{2}{S^2}\frac{\partial e_{S\theta}}{\partial\theta_1}+\frac{\partial^2 e_\theta}{\partial S^2}+\frac{1}{S^2}\frac{\partial^2 e_S}{\partial\theta_1^2}+\frac{2}{S}\frac{\partial e_\theta}{\partial S}-\frac{1}{S}\frac{\partial e_S}{\partial S} =\frac{1}{S^4}\left(\frac{\partial w}{\partial\theta_1}\right)^2-\frac{2}{S^3}\frac{\partial w}{\partial\theta_1}\frac{\partial^2 w}{\partial S\partial\theta_1}-\frac{1}{S^2}\left[\frac{\partial^2 w}{\partial S^2}\frac{\partial^2 w}{\partial\theta_1^2}-\left(\frac{\partial^2 w}{\partial S\partial\theta_1}\right)^2\right] -\frac{1}{S}\frac{\partial w}{\partial S}\frac{\partial^2 w}{\partial S^2} \quad (2)$$

where w is the displacement and M_S, M_θ, $M_{S\theta}$ represent moment resultants and the mass density per unit length defined as

$$\rho_1=\int_{-0.5}^{0.5}\rho(Z)dZ \quad (3)$$

The force and moment resultants are expressed by [39,41]:

$$[(N_S,N_\theta,N_{S\theta}),(M_S,M_\theta,M_{S\theta})]=\int_{-h/2}^{h/2}(\sigma_S,\sigma_\theta,\sigma_{S\theta})[1,z]dz \quad (4)$$

The stress–displacement relations for non-homogeneous orthotropic truncated conical shells are given as follows:

$$\begin{bmatrix}\sigma_S\\ \sigma_\theta\\ \sigma_{S\theta}\end{bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0\\ Q_{21}&Q_{22}&0\\ 0&0&Q_{66}\end{bmatrix}\begin{bmatrix}e_S-Z\frac{\partial^2 w}{\partial S^2}\\ e_\theta-Z\left(\frac{1}{S^2}\frac{\partial^2 w}{\partial\theta_1^2}+\frac{1}{S}\frac{\partial w}{\partial S}\right)\\ e_{S\theta}-Z\left(\frac{1}{S}\frac{\partial^2 w}{\partial S\partial\theta_1}-\frac{1}{S^2}\frac{\partial w}{\partial\theta_1}\right)\end{bmatrix} \quad (5)$$

where e_S, e_θ, $e_{S\theta}$ are the strains on the reference surface and the quantities Q_{ij}, (i,j = 1, 2, 6) are

$$Q_{11}=\frac{E_{01}\varphi_1(Z)}{1-v_{12}v_{21}},\quad Q_{22}=\frac{E_{02}\varphi_1(Z)}{1-v_{12}v_{21}},\quad Q_{12}=Q_{21}=v_{21}Q_{11} =v_{12}Q_{22},\quad Q_{66}=2G_0\varphi_1(Z) \quad (6)$$

in which v_{12} and v_{21} are the Poisson's ratios, assumed to be constant and satisfying $v_{21}E_{01}=v_{12}E_{02}$ [42].

Based on the above relationships, the non-linear motion and compatibility Eqs. (6) and (7) may be written in the form as:

$$\begin{aligned}Q(x,\theta,t)\equiv{}&\delta_1 e^{2x}\frac{\partial^4\Psi_1}{\partial x^4}+\delta_2 e^{2x}\frac{\partial^3\Psi_1}{\partial x^3}+(\delta_3+S_1e^x\cot\gamma)e^{2x}\frac{\partial^2\Psi_1}{\partial x^2}\\ &+(\delta_4+3S_1e^x\cot\gamma)e^{2x}\frac{\partial\Psi_1}{\partial x}+2S_1e^{3x}\Psi_1\cot\gamma+\delta_5e^{2x}\frac{\partial^4\Psi_1}{\partial\theta_1^4}\\ &+\delta_6e^{2x}\frac{\partial^4\Psi_1}{\partial x^2\partial\theta_1^2}+\delta_7e^{2x}\frac{\partial^3\Psi_1}{\partial x\partial\theta_1^2}+\delta_8e^{2x}\frac{\partial^2\Psi_1}{\partial\theta_1^2}-\delta_9\frac{\partial^4 w}{\partial\theta_1^4}\\ &-\delta_{10}\frac{\partial^4 w}{\partial x^2\partial\theta_1^2}+\delta_{11}\frac{\partial^3 w}{\partial x\partial\theta_1^2}-\delta_{12}\frac{\partial^2 w}{\partial\theta_1^2}-\delta_{13}\frac{\partial^4 w}{\partial x^4}\\ &+\delta_{14}\frac{\partial^3 w}{\partial x^3}-\delta_{15}\frac{\partial^2 w}{\partial x^2}+\delta_{16}\frac{\partial w}{\partial x}\\ &+e^{2x}\left(\frac{\partial^2\Psi_1}{\partial\theta_1^2}+\frac{\partial\Psi_1}{\partial x}+2\Psi_1\right)\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial w}{\partial x}\right)\\ &+e^{2x}\left(\frac{\partial^2\Psi_1}{\partial x^2}+3\frac{\partial\Psi_1}{\partial x}+2\Psi_1\right)\left(\frac{\partial^2 w}{\partial\theta_1^2}+\frac{\partial w}{\partial x}\right)\\ &-2e^{2x}\left(\frac{\partial^2\Psi_1}{\partial x\partial\theta_1}+\frac{\partial\Psi_1}{\partial\theta_1}\right)\left(\frac{\partial^2 w}{\partial x\partial\theta_1}-\frac{\partial w}{\partial\theta_1}\right)\\ &-\rho_1hS_1^4e^{4x}\frac{\partial^2 w}{\partial t^2}=0\end{aligned} \quad (7)$$

$$\begin{aligned}&\Delta_1e^{2x}\frac{\partial^4\Psi_1}{\partial x^4}+\Delta_2e^{2x}\frac{\partial^3\Psi_1}{\partial x^3}+\Delta_3e^{2x}\frac{\partial^2\Psi_1}{\partial x^2}+\Delta_4e^{2x}\frac{\partial\Psi_1}{\partial x}+\Delta_5e^{2x}\\ &\times\frac{\partial^4\Psi_1}{\partial x^2\partial\theta_1^2}+\Delta_6e^{2x}\frac{\partial^3\Psi_1}{\partial x\partial\theta_1^2}+\Delta_7e^{2x}\frac{\partial^2\Psi_1}{\partial\theta_1^2}+\Delta_8e^{2x}\frac{\partial^4\Psi_1}{\partial\theta_1^4}-\Delta_9\\ &\times\frac{\partial^4 w}{\partial\theta_1^4}+\Delta_{10}\frac{\partial^4 w}{\partial x^2\partial\theta_1^2}+\Delta_{11}\frac{\partial^3 w}{\partial x\partial\theta_1^2}+\Delta_{12}\frac{\partial^2 w}{\partial\theta_1^2}-\Delta_{13}\frac{\partial^4 w}{\partial x^4}+\Delta_{14}\\ &\times\frac{\partial^3 w}{\partial x^3}+(\Delta_{15}+S_1e^x\cot\gamma)\frac{\partial^2 w}{\partial x^2}+(\Delta_{16}-S_1e^x\cot\gamma)\frac{\partial w}{\partial x}\\ &+\left(\frac{\partial w}{\partial\theta_1}\right)^2+2\frac{\partial w}{\partial\theta_1}\frac{\partial^2 w}{\partial x\partial\theta_1}-\left(\frac{\partial w}{\partial x}-\frac{\partial^2 w}{\partial x^2}\right)\frac{\partial^2 w}{\partial\theta_1^2}-\left(\frac{\partial^2 w}{\partial x\partial\theta_1}\right)^2\\ &-\left(\frac{\partial w}{\partial x}-\frac{\partial^2 w}{\partial x^2}\right)\frac{\partial w}{\partial x}\\ &=0\end{aligned} \quad (8)$$