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Effect of thermal wave propagation on thermoelastic behavior of functionally graded materials in a slab symmetrically surface heated using analytical modeling

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ABSTRACT

Design and development of FGMs as the heat treatable materials for high-temperature environments with thermal protection require understanding of exact temperature and thermal stress distribution in the transient state. This information is primary tool in the design and optimization of the devices for failure prevention. Frequently FGMs are used in many applications that presumably produce thermal energy transport via wave propagation. In this study, transient non-Fourier temperature and associated thermal stresses in a functionally graded slab symmetrically heated on both sides are determined. Hyperbolic heat conduction equation in terms of heat flux is used for obtaining temperature profile. Separation of variables scheme based on the variation of parameters is implemented to solve the non-homogenous thermoelastic problem in which any arbitrary temperature distribution can be employed. Physical properties are assumed to vary exponentially in the media and the problem is analyzed for different mechanical boundary conditions. Furthermore effect of the material inhomogeneity, thermal relaxation time and the Fourier number on the stress distribution, temperature variation and jumps is investigated. Parallel computation is used to present the fully converged numerical results. Findings indicate the non-Fourier heat conduction has significant influence on the dynamic temperature and stress field. Based on the results it is suggested that in the design of FG structures against failure under thermal loading and heat treatability, the hyperbolic model is more appropriate than the classical Fourier model.

1. Introduction

Functionally graded materials (FGMs) are modern inhomogeneous materials in which two or more different material ingredients changes continuously and gradually resulting in elimination of interface problems and consequently uniformity of the stress and temperature distributions. FGMs are especially introduced to significantly reduce the temperature and thermal stresses on structures at sever thermal loading. Frequently, these materials are made from ceramics and metals, with continuous change in composition in the benefit of the attractive features of the constituents. Ceramics are beneficial in high compressive strength and temperature regions due to their higher thermal resistance characteristics, while suffers from low fracture toughness. On the contrary, metals exhibits better tensile strength and thermal shock resistance but cannot withstand exposure to high temperatures. However, FGMs withstand extremely high-temperature and high-thermal shocks environments while maintain their structural integrity. Thus, these materials have been widely utilized in many applications such as heat engine components, wear resistant linings, thermal shields, laser applications and fusion reactors plasma facings. In addition, it is well known that thermal stress distributions in a transient state can vary sharply compared with the one in a steady state. Hence, to design a reliable FGM structure working under severe thermal loadings it is crucial to determine exact transient temperature bearing. Among the issues relevant to FGMs, the heat transfer problem is typically one-dimensional in many applications and many types of FGMs such as those that called thermal barrier coatings (TBCs).

There are different theories relevant to heat conduction in solids. The classic parabolic heat conduction theory is based on the Fourier law as the constitutive relation. Fourier law implicitly postulates that the propagation speed of thermal disturbances is infinite which is in contrast to what occurs in reality. However, this law gives quite reliable results in many practical situations, but it breaks down for materials in situations involving extreme temperature gradients, low temperatures and conduction at very short periods of time such as pulsed-laser processing of metals and

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semiconductors, film applications, laser surgery, etc. where the typical response time is in the order of picoseconds and consequently the wave nature of thermal propagation becomes dominant. In addition Fourier law is ineffective at the very small length scales associated with small-scale systems (e.g., nano- or micro-scale systems). In these applications the results predicted by using the Fourier heat conduction model differ significantly from the experimental results [1,2]. The most frequently used model for these problems is the hyperbolic heat conduction (HHC) that is special case of the dual-phase-lagging conduction equation. It is based on the constitutive relation introduced by Catteneo [3] and Vernotte [4] and developed by Tzou [5]. This model takes into account a finite propagation speed for thermal disturbances by considering a thermal relaxation time as a material property. Hence it involves high heat flux and strong unsteadiness. While high thermal loading such as high heat flux is applied to FGMs, hyperbolic heat conduction theory is preferable to study the thermal behavior of these materials.

Research on composites and FGMs have been widely progressed in the stress, buckling, vibration and post-buckling analysis of a structure under the thermo-mechanical conditions using analytical and numerical methods. For instance, Asghari et al. [6] analytically investigated the size-dependent static and vibration behavior of micro-beams made of FGMs on the basis of the modified couple stress theory in the elastic range to alleviate the shortcomings caused by the small scale systems. Further, Kargarnovin et al. [7] presented a rather new semi-analytical method towards investigating the free vibration analysis of generally laminated composite beam (LCB) with a delamination. Among the fields relevant to FGMs, thermal stress problems occur in many branches of engineering and have received considerable attention both in analysis and design [8–11]. In most of them the analyses are based on the Fourier heat conduction theory, due to its simplicity. Ref. [12] presents basic concepts and principles to advanced levels of thermal stress analysis in various structures made of homogenous and FG materials using coupled and uncoupled thermoelsticity theories and further analytical schemes are used to solve different problems. Moreover, in other works several strategies have been applied to solve thermoelastic models on FGMs analytically. However, there are few works of the exact analytical treatments for the transient thermoelastic problems of FGMs. In addition, the properties such as thermal expansion are assumed constant mostly. Aboudi et al. [13] presented a micromechanical theory for the response of functionally graded metal-matrix composites subjected to thermal gradients. The outlined approach is useful in generating favorable stress distributions in the presence of thermal gradients by appropriately grading the internal microstructural details of the composite. Vel and Batra [14] analyzed the three-dimensional transient thermal stresses of the functionally graded rectangular plate by extending the analytical technique as a power series method. Ohmichi and Noda [15] obtained plane thermal stresses in a functionally graded plate (FGP) subjected to a partial heating by analytical solution. An analytical description of thermal-stress states in a functionally graded plate subjected to through-thickness heat flow is presented by Tsukamoto [16] via combining micromechanical with macromechanical approaches. Ootao and Tanigawa [17–19] obtained a three-dimensional solution for transient thermal stresses of functionally graded rectangular plate with exponentially variation properties due to non-uniform and partial heat supply. They used a combined method of Laplace and finite cosine transformations. Furthermore, similar method was employed for theoretical transient thermal stress analysis of the functionally graded magneto-electro-thermo-elastic strip due to unsteady and non-uniform surface heating by Ootao et al. [20]. Nonlinear thermal bending analysis was presented for a simply supported, shear deformable functionally graded plate without or with piezoelectric actuators subjected to the combined action of thermal and electrical loads by Shen [21]. He indicated that the temperature-dependency of FGMs cannot be neglected in the thermal bending analysis. Sharifishourabi et al. [22] studied the effects of material gradation on thermo-mechanical stresses in functionally graded Euler–Bernoulli beams and found it has considerable effect on the neutral axis position. Malekzadeh and Shojaee [23] investigated the effect of different parameters on the dynamic response of functionally graded (FG) plates under a moving heat source using a three-dimensional finite element method.

Research on transient thermoelastic behavior of FGMs in which temperature only effect on stress field but not vice versa are divided into three categories. In the first category, thermal diffusion term is neglected in the heat conduction equation [24]. They believe for common materials the ratio value of stress wave propagation time to thermal diffusion time is of the order 10^{-6}, so such an assumption is reasonable. However, it means temperature variation by space has no significant effect on thermal stresses. It seems this is not to be according to practical problems. In the second category, some of researchers obtained transient space dependent temperature field, but neglected the term, which contains acceleration in the equation of motion for thermoelastic investigation [18,19,25]. It means stress wave propagation speed do not affect stress filed that is a limited theory. In the third category, some of other researchers considered all transient terms on the study of transient thermoelastic behavior of various structures [26]. In these works effects of material properties and dimensions on stress variations are less investigated especially when the wave nature of thermal energy transport becomes dominant.

A few investigations on uncoupled thermal stresses in thermoelastic homogenous materials have been made using the hyperbolic heat conduction model. Manson and Rosakis [27,28] developed a solution for the hyperbolic heat conduction equation for a travelling point heat source around a propagating crack tip, and also experimentally measured the temperature distribution at the tip of the dynamically propagating crack. Chen and Hu [29] obtained the transient thermal stresses around a crack in a thermo-elastic half-plane under a thermal shock using the hyperbolic heat conduction theory. They applied Fourier, Laplace transforms and singular integral equations to solve the temperature and thermal stress fields consecutively. Wang et al. [30] developed a finite element/finite difference scheme for the non-classical heat conduction and associated thermal stresses in the materials with temperature dependent properties. They found the obtained thermal stresses are considerably difference from those associated with classical heat conduction theory. In addition, some researchers considered finite wave propagation speed in coupled themoelasticity of materials using second-order derivative terms with respect to time and solved coupled energy equation using numerical and semi analytical techniques. In this regard, Sumi [31] performed a numerical analysis for two-dimensional thermo-mechanical wave propagations in the homogenous thermoelastic solids using generalized coupled theory. The coupled thermoelasticity problem with no energy dissipation in functionally graded thick hollow cylinder was studied and developed using Green–Naghdi theory by Hosseini [32]. In this work, the calculations of the mechanical displacements, temperature distribution and thermal stresses have been provided by a hybrid numerical solution. Furthermore, the cylinder is assumed to be multilayer cylinder across the thickness and material properties in each layer are assumed to be constant. In addition the thermal relaxation time as a property of FGM was not considered in this work. Shariyat [33] performed a nonlinear generalized and classical thermoelasticity analyses for functionally graded thick cylinders with temperature-dependent material properties subjected to