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Merit functions for the linac optics design for colliders and light sources

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ABSTRACT

Optics matching and transverse emittance preservation are key goals for a successful operation of modern high brightness electron linacs. The capability of controlling them in a real machine critically relies on a properly designed magnetic lattice. Conscious of this fact, we introduce an ensemble of optical functions that permit to solve the often neglected conflict between strong focusing, typically implemented to counteract coherent synchrotron radiation and transverse wakefield instability, and distortion of the transverse phase space induced by chromatic aberrations and focusing errors. A numerical evaluation of merit functions is applied to existing and planned linac-based free electron lasers.

1. Introduction

State-of-the-art linear colliders and x-ray free electron lasers (FELs) require electron beams with high brightness (10^9–10^{12} A/mm^2) and small longitudinal emittance ($\sim 10^{-2}$ MeV ps) [1]. The charge density is high enough to drive collective effects (wakefields) that, notwithstanding the high beam rigidity at energies up to the GeV range, may increase the six-dimensional emittance relative to the injection level [2]. In the transverse planes, the wakefields can be counteracted by imposing some constraints to the optics design, *e.g.*, to avoid beam break up along the accelerator [3–5] and to minimize coherent synchrotron radiation (CSR) effects in magnetic compressors [6–11]. Additional constraints are typically due to the optimization of the performance of diagnostics [12,13] and beam collimation [14,15]. Strong focusing is typically prescribed for all the aforementioned cases, which has the additional advantage of cumulating the desired betatron phase advance in short distances, so as to save space and finally minimize the cost of the facility. Unfortunately, it may also hamper the main goal of emittance preservation through the excitation of optical aberrations and potentially leads, through focusing errors, to beam optics mismatch that can in turn corrupt the optics scheme adopted for the suppression of the instabilities. Based on these often conflicting requirements and on the partial lack, in the archival literature, of established strategies to design and optimize the optics of linear colliders and fourth generation light sources, we introduce an ensemble of merit functions that offer, to our knowledge for the first time, a guidance to the definition of quadrupole strengths and Twiss functions in high brightness electron linacs. This article is organized as follows. In Section 2, optics sensitivity to focusing errors is defined. When applied to linacs that contain magnetic bunch length compression, it is used to evaluate final optics mismatch and chromatic emittance dilution in one- *versus* multi-stage compression scheme. In Section 3, we provide prescriptions for the design of the chromatic *H*-function in a magnetic compressor in order to balance transverse and longitudinal perturbations excited by CSR, namely, emittance growth and microbunching instability [16–18]. In Section 4, local chromaticity is introduced. It is used to show that, in spite of their weak transverse impedance, superconducting linacs may still require rather strong focusing (as is customary in normal conducting linacs) in order to avoid troublesome optics mismatch. Conclusions are given in Section 5.

2. Beam optics mismatch and emittance growth

2.1. Optics sensitivity to focusing errors

Our treatment basically applies well known concepts of linear and nonlinear accelerator physics. Some of them however, like the tune-shift-with-amplitude and the chromaticity, are not suited to describe local sources of phase space distortion. This fact justifies a less conventional approach in which we define a sensitivity coefficient that identifies sources of optics mismatch along the lattice. We adopt the definition of the mismatch parameter introduced in [19]. In addition, we assume a relative focusing error $k\tau$, due to a perturbation τ in the quadrupole strengths, so that the final mismatch parameter is computed in each plane as follows:

$$m_\delta^f \cong 1+\frac{1}{2}\left[\left(\sum_{i=1}^{N} k_i\beta_i L_i\tau_i \cos(2\Delta\mu_i)\right)^2+\left(\sum_{i=1}^{N} k_i\beta_i L_i\tau_i \sin(2\Delta\mu_i)\right)^2\right] \quad (1)$$

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The sum in Eq. (1) is over N quadrupoles, L is the quadrupole magnetic length, k its strength, τ the fractional strength error and $\Delta\mu$ the betatron phase advance. We now evaluate Eq. (1) one quadrupole at the time, while keeping all other magnets at their nominal strength ($\tau=0$). We thus obtain the mismatch induced at the *end* of the line by each *individual* quadrupole magnet:

$$m_\delta^q - 1 \cong \frac{1}{2}(k\beta L\tau)_q^2 \equiv \xi_q(\tau) \quad (2)$$

We define ξ_q as the optics sensitivity to focusing errors. If we assume that the mismatch parameter at the end of the line, $m_\delta^f = 1+C$, is the result of many identical, uncorrelated and small contributions $\xi_q \ll 1$, the maximum sensitivity allowed to each quadrupole (tolerance) is of the order of $m_\delta^q - 1 = \xi_q \approx C/\sqrt{N}$. As an example, for $C=5\%$ and $N=100$, lattice regions characterized by $\xi_q \geq 0.5\%$ should be deemed worthy of stricter tolerances in strength value or in need of weaker focusing. It can be shown through the beam matrix formalism that Eq. (2) also describes the final relative emittance growth induced by each individual error kick.

2.2. Applicability

This section shows the application of the method with an example. Fig. 1 shows two solutions for the main linac optics design of FERMI@Elettra FEL [20]; one is the baseline adopted in the first stage of commissioning [21], the other is the result of a subsequent optimization carried out in order to minimize m_δ^q at the quadrupole locations. This optimized configuration eventually led to the first successful lasing [22,23]. Fig. 2 compares the sensitivity to mean energy error and to energy spread for the two scenarios. Unlike the baseline design, the optimized lattice pushes the sensitivity to mean energy error below the tolerance of 0.6% ($C=5\%$ and $N=74$). Chromatic sensitivity is reduced almost everywhere: a final emittance growth smaller than 1% in the horizontal plane and 2% in the vertical, *i.e.* a factor 2 smaller than in the initial lattice configuration, is predicted. In both scenarios, most of the chromatic emittance dilution is expected to happen right downstream the first compressor area (BC1). However, the optimized optics design reduced the beam projected emittance measured in this region from values typically larger than 1.0 mm mrad for 350 pC beam charge [22] to 0.85 mm mrad for 500 pC [24]. These experimental values were obtained with an uncompressed beam, so that CSR effects can be excluded and the improved emittance control can reasonably be attributed to the improved optics design.

In Fig. 2, major sources of chromatic emittance dilution are indicated by the highest sensitivity value (bottom row). This is particularly large in proximity of the chicane for magnetic bunch length compression. To understand this fact, we invite to compare Fig. 2 with Fig. 3, which shows the chromatic sensitivity computed for the PAL XFEL baseline design [25]. Both in FERMI and PAL XFEL, the highest sensitivity is in proximity of the first magnetic compressor, where the *relative* energy spread is the largest over the entire linac by virtue of the low beam energy and the energy chirp imparted to the beam for compression. The lower sensitivity of the PAL XFEL with respect to FERMI is attributed to the PAL XFEL multi-stage compression. In fact, a multi-stage scheme can be arranged in a way that requires smaller energy spread in the first compressor and, accordingly, weaker focusing to counteract CSR induced emittance growth. This item is discussed in detail in the next Section.

3. Optics control in magnetic compressors

3.1. Chromatic H-function

CSR-induced phase space distortion in magnetic compressors is a critical aspect of any high brightness linac design. CSR-induced emittance growth is primarily due to betatron oscillations of the longitudinal slices of the bunch around different dispersive trajectories. Even in the case of achromatic lines, the increased betatron invariant of the slices' centroid is not recovered as the dispersion function collapses to zero [26]. If we describe the CSR effect in a short magnet with the single-kick approximation, we are allowed to use the beam matrix formalism to obtain a simple expression for the projected emittance growth:

$$\varepsilon \cong \left[\det\begin{pmatrix} \varepsilon_0\beta+\eta^2\sigma^2_{\delta,CSR} & -\varepsilon_0\alpha+\eta\eta'\sigma^2_{\delta,CSR} \\ -\varepsilon_0\alpha+\eta\eta'\sigma^2_{\delta,CSR} & \varepsilon_0\frac{1+\alpha^2}{\beta}+\eta'^2\sigma^2_{\delta,CSR} \end{pmatrix}\right]^{1/2} = \varepsilon_0\sqrt{1+\frac{H}{\varepsilon_0}\sigma^2_{\delta,CSR}}, \quad (3)$$

where ε and ε_0 are the geometric RMS emittance after and before the CSR kick, respectively, $H=[\eta^2+(\beta\eta'+\alpha\eta)^2]/\beta$ in the bending plane, η is the energy dispersion function, η' is its first derivative with respect to the curvilinear longitudinal coordinate s and $\sigma_{\delta,CSR}$ is the CSR-induced RMS relative energy spread [9]. Since $\sigma_{\delta,CSR}$ is inversely proportional to the bunch length, the transverse CSR effect in a four dipoles magnetic chicane is dominated by the radiation emission in the second half of the system, where the bunch reaches its shortest duration. We consider typical values in the compressor such as bending angle $\theta \leq 0.1$ rad, dipole length $l \approx 0.3$ m, $\eta \leq 0.2$ m and $\beta \geq 3$ m. We assume a beam waist in the second half of the chicane and β that remains in the range 3–5 m across it. Then, $H \approx \eta^2/\beta \approx 0.01\ m$ at the end of the 3rd dipole and $H \approx \beta\eta'^2 \approx \beta\theta^2 \approx 0.03\ m$ at the entrance of the 4th dipole. CSR contribution from 3rd dipole is actually suppressed with respect to the 4th because the beam reaches its final shortest length only at the very *end* of that magnet. Hence, $\sigma_{\delta,CSR}$ induced in the 3rd dipole can be up to $\sim C$ times smaller than in the 4th dipole, where C is the compression factor. In conclusion, Eq. (3) prescribes to shrink the *H*-function at the entrance of the fourth dipole in order to suppress

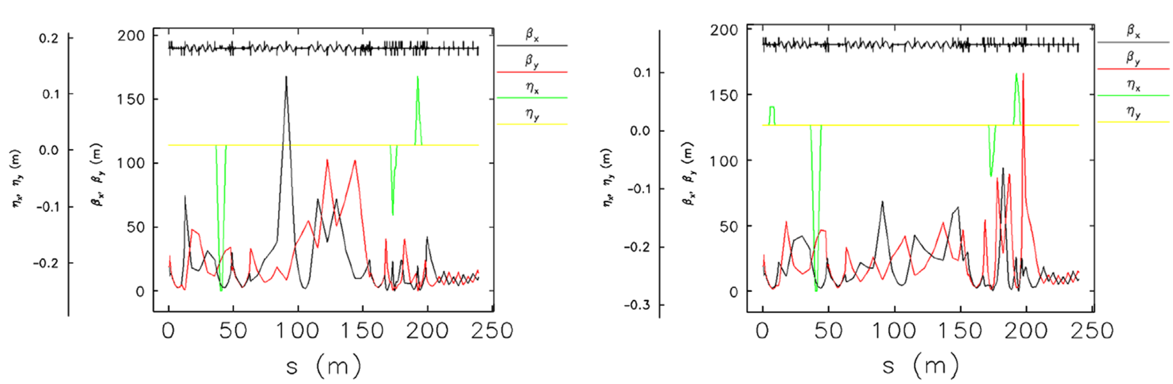

Fig. 1. Optics in the FERMI main linac (from injector end to undulator end) for the baseline (left) and optimized design (right).