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# A method for fast feature extraction in threshold scans

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ABSTRACT

We present a fast, analytical method to calculate the threshold and noise parameters from a threshold scan. This is usually done by fitting a response function to the data which is computationally very intensive. The runtime can be minimized by a hardware implementation, e.g. using an FPGA, which in turn requires to minimize the mathematical complexity of the algorithm in order to fit into the available resources on the FPGA. The systematic errors of the method are analyzed and reasonable choices of the parameters for use in practice are given.



## 1. Introduction

When characterizing detector readout electronics it is a common task to determine its threshold and noise. This is typically done by performing a so-called threshold scan where, e.g. in case of a silicon detector, a known charge is injected and the probability for the detector to respond is recorded as a function of the injected charge. The resulting response function (1) is a convolution of the pure threshold (step function) and a Gaussian noise function. The traditional method to obtain the threshold and noise is to fit Eq. (1) to the data. The disadvantage of this approach is that it usually involves solving a nonlinear minimization problem with the following steps required in the least squares approach:

- finding a suitable choice of starting values, range and step sizes to vary the free parameters,
- calculating the response function, in this case it involves calculating the non-trivial error function,
- determination and minimization of the residuals by varying the free parameters.

When a hardware implementation, e.g. a Field Programmable Gate Array (FPGA), is desired, the calculation of complex mathematical functions and decisions such as the optimized values for the parameters is usually avoided. In FPGAs, elementary logical and mathematical operations such as additions and shifting are much more efficient. In order to execute more complex mathematical operations, modern FPGAs often have special circuitry, e.g. efficient multiplication [2]. However, such resources are limited.

In the following, we present an alternative method for fast extraction of the relevant parameters from the data of a threshold scan. Our strategy is to simplify the problem mentioned above, such that the optimization problem can be replaced by a direct analytical calculation.

## 2. Fast feature extraction

Fig. 1 shows a data sample which was recorded during a threshold scan of an ATLAS FE-I3 [3] type frontend module. The charge $q_i$ to be injected was chosen by setting a Digital-Analog-Converter (DAC) to values from 270 to 350 where the amount of charge is a linear function of the DAC-value. Each charge $q_i$ was injected 100 times and the number $R(q_i)$ of hits detected by the frontend module was recorded. The response function shows the typical S-like shape.

It can be seen that the total number of recorded responses in a threshold scan is equivalent to the integral over the response function within the probed range. Since this number is very easy to obtain (i.e. count) when carrying out a threshold scan, it is a suitable value for an analytic determination of the threshold from the data. The noise can be determined analytically with a similar approach. In the following, the required equations for these calculations are derived.

### 2.1. Derivation

Fig. 2 illustrates the chosen nomenclature and integration ranges. Labeling the integrals over a limited range of the response

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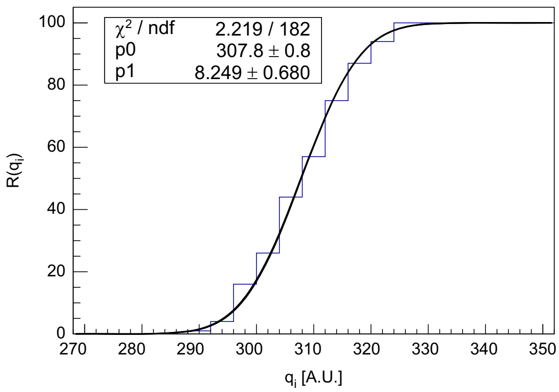


**Fig. 1.** Number of measured responses of the ATLAS FE-I3 frontend module for 100 injections at each value of the input charge $q_i$ with a step size $d=4$ [A.U.]. The two curves that are virtually indistinguishable are plotted over the data and show the response functions as obtained by a fit to the data ($\mu = 307.8 \pm 0.8$, $\sigma = 8.25 \pm 0.68$) and by the method presented in this paper ($\mu = 307.84$, $\sigma = 8.172$).

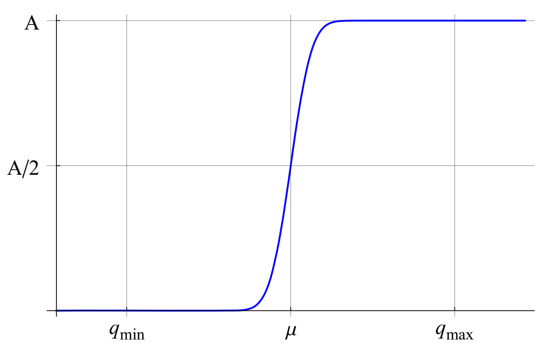


**Fig. 2.** Error function with integration limits $q_{min}$ and $q_{max}$, threshold $\mu$ and $A$ injections with symmetry point $(\mu, A/2)$. Figure from Ref. [1].

function (1) as $\tilde{M}$ and $\tilde{m}_\mu$, we get

$$R(q) = \frac{A}{2} \cdot \left(1 + \text{erf}\left(\frac{q-\mu}{\sqrt{2} \cdot \sigma}\right)\right) \tag{1}$$

$$\tilde{M} = \int_{q_{min}}^{q_{max}} R(q)\, dq \tag{2}$$

$$\tilde{m}_\mu = \int_{q_{min}}^{\mu} R(q)\, dq + \int_{\mu}^{q_{max}} A - R(q)\, dq \tag{3}$$

From the symmetry of $R(q)$ it follows:

$$\tilde{M} = (q_{max} - \mu) \cdot A \tag{4}$$

In principle one must assert that $q_{max} - \mu = \mu - q_{min}$. However, in practice this constraint can be omitted because for sufficiently small $q_{min}$, i.e. an injected charge sufficiently far below the threshold $\mu$, no detector response will be recorded. Thus, these points do not contribute to the value of the integral which means that any deviation from the symmetry in that region will have no influence on the result. The very same argument can be made for the points far above the threshold, where the detector will always respond. From Eq. (4) we obtain the following formula to calculate the threshold:

$$\mu = q_{max} - \frac{\tilde{M}}{A} \tag{5}$$

With the definite integral

$$\int_0^x \text{erf}(t)\, dt = x\, \text{erf}(x) + \frac{1}{\sqrt{\pi}}(e^{-x^2} - 1) \tag{6}$$

of the error function (see e.g. Eq. (8.100d) in [4]) we get

$$\int_{-\infty}^{\mu} R(q)\, dq = \int_{-\infty}^{\mu} \frac{A}{2} \cdot \left(1 + \text{erf}\left(\frac{q-\mu}{\sqrt{2} \cdot \sigma}\right)\right) dq$$
$$= \frac{A \cdot \sigma}{\sqrt{2\pi}} = \lim_{q_{min} \to -\infty} \int_{q_{min}}^{\mu} R(q)\, dq \tag{7}$$

Due to the symmetry of $R$ it also follows that

$$\int_{\mu}^{\infty} A - R(q)\, dq = \frac{A \cdot \sigma}{\sqrt{2\pi}} = \lim_{q_{max} \to \infty} \int_{\mu}^{q_{max}} A - R(q)\, dq \tag{8}$$

Using Eqs. (7) and (8) in Eq. (3), $\tilde{m}_\mu$ can be written as

$$\tilde{m}_\mu = \lim_{q_{min} \to -\infty} \int_{q_{min}}^{\mu} R(q)\, dq + \lim_{q_{max} \to \infty} \int_{\mu}^{q_{max}} A - R(q)\, dq = \frac{2 \cdot A \cdot \sigma}{\sqrt{2\pi}} \tag{9}$$

For sufficiently small $q_{min}$ and sufficiently large $q_{max}$ we can omit the limits in Eq. (9). Solving for $\sigma$, we obtain

$$\sigma = \frac{\tilde{m}_\mu}{A} \cdot \sqrt{\frac{\pi}{2}} \tag{10}$$

The justification of sufficiently large integration ranges can be made similarly to the considerations for Eq. (4). Writing down the relative deviation $\zeta$ caused by omitting the limit in Eq. (9), we obtain

$$\zeta = 1 - \frac{\int_{-k \cdot \sigma}^{\mu} R(q)\, dq + \int_{\mu}^{k \cdot \sigma} A - R(q)\, dq}{\int_{-\infty}^{\mu} R(q)\, dq + \int_{\mu}^{\infty} A - R(q)\, dq}$$
$$= e^{-k^2/2} - k \cdot \sqrt{\frac{\pi}{2}} + k \cdot \sqrt{\frac{\pi}{2}} \cdot \text{erf}\left(\frac{k}{\sqrt{2}}\right) \tag{11}$$

In order to simplify the notation, the integration ranges are written as $k$ multiples of $\sigma$. Note that $\zeta$ is a function of only $k$. Fig. 3 shows that the relative deviation already for a range of $6 \cdot \sigma$ around the threshold is smaller than $4 \times 10^{-10}$, which is negligible compared to the maximum precision of 9 decimal digits for operations on a 32 bit single precision floating point number.

We now replace the integrals (2) and (3) with their respective sums according to the Riemann's interpretation of integrals (see e.g. [5]):

$$\lim_{m \to \infty} \sum_{i=0}^{m-1} f(\tau_i) \cdot (t_{i+1} - t_i) = \int_a^b f(t)\, dt, \quad \text{with } t_0 = a,\ t_m = b \tag{12}$$

When using $n = m$ equidistant measurements with a distance $t_{i+1} - t_i = d$ and choosing $q_i = \tau_i = (t_{i+1} + t_i)/2$, Eq. (12) can be written as

$$d \cdot \lim_{m \to \infty} \sum_{i=0}^{m-1} f(\tau_i) = \int_a^b f(t)\, dt, \quad \text{with } d = \frac{b-a}{m} \tag{13}$$

This choice of $q_i$ results in each measurement location being centered between the limits of its respective interval. As $f(t)$ is

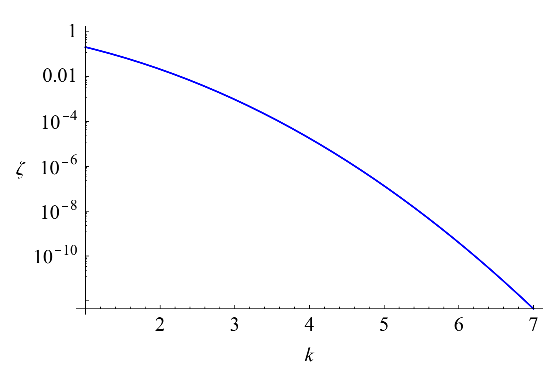


**Fig. 3.** Relative deviation between the values of the integral $\tilde{m}_\mu$ over the response function when integrating from $-\infty$ to $\infty$ compared to using the finite limits $\pm k \cdot \sigma$. Figure from Ref. [1].