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Analysis of thick isotropic and cross-ply laminated plates by generalized differential quadrature method and a Unified Formulation

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ABSTRACT

In this paper, the Carrera Unified Formulation and the generalized differential quadrature technique are combined for predicting the static deformations and the free vibration behavior of thin and thick isotropic as well as cross-ply laminated plates. Through numerical experiments, the capability and efficiency of this technique, based on the strong formulation of the problem equations, are demonstrated. The numerical accuracy and convergence are also examined. It is worth noting that all the presented numerical examples are compared with both literature and numerical solutions obtained with a finite element code. The proposed methodology appears to be able to deal not only with uniform boundary conditions, such as fully clamped or completely simply-supported, but also with mixed external conditions, that can be clamped, supported or free.

1. Introduction

In this paper the Unified Formulation (UF) proposed by Carrera et al. [1–5] is used to derive the equations of motion and boundary conditions and to analyze isotropic and cross-ply laminated plates by the generalized differential quadrature (GDQ) method. A higher order theory (HSDT) as proposed before by Kant et al. [6,7] considering non-zero normal deformation ε_z is adopted, and an expanded higher-order shear deformation up to the cubic expansion in z for all in-plane displacement components is worked out.

The combination of the UF and collocation with the GDQ method provides an easy, highly accurate framework for the solution to plates, under any kind of shear deformation theory, irrespective of the geometry, loads or boundary conditions. In this sense, this methodology can be considered a generalized UF.

Many shear deformation theories, that involve a constant transverse displacement across the thickness direction and make the transverse normal strain and stress negligible, were proposed. This assumption is adequate for thin-plates or plates for which the thickness-to-side h/a is smaller than 0.1. For higher h/a ratios, the use of shear deformation theories including the contribution of the transverse normal strain and stress is fundamental. Among such theories, the pioneering higher-order plate theory by Lo et al. [8,9] or the ones by Kant and colleagues [6,7] can be cited. Recently, the works by Batra and Vidoli [10] and Carrera [1,2,11] show interesting ways of computing transverse and normal stresses in laminated composite or sandwich plates. Higher-order theories in the thickness direction were also addressed by Librescu et al. [12], Reddy [13] and more recently by Fiedler and colleagues [14], who considered polynomial expansions in the thickness direction. None of such approaches carried out the analysis by the GDQ method. Some other HSDTs have been presented over the years regarding composite materials, FGMs as well as beams, plates and shells [15–19], however it is impossible to cite them all. The use of alternative methods, such as meshless methods based on generalized differential quadrature, is attractive due to the absence of a mesh and the use of strong-form methods. The present work adds some numerical applications and results to the vast bibliography concerning meshless methods [20–45].

In this paper, it is investigated for the first time how the UF by Carrera can be combined with the GDQ method for treating thick isotropic and cross-ply laminated plates, using a refined higher-order shear and normal deformation theory. The quality of the present method in predicting static deformations, and free vibrations of thick isotropic and cross-ply laminated plates is compared and discussed with other methods in some numerical examples.

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2. Fundamental equations via Unified Formulation

Many details of the Unified Formulation (UF) by Carrera can be inspected in [1–4,11]. In the present section the equations of motion and the corresponding boundary conditions are worked out by using the fundamental nuclei [1–4,11] and a compact matrix form notation.

2.1. Displacement field

By defining a displacement field, that involves all layers, an equivalent single-layer theory is proposed. The displacement field is expressed as

$$\begin{aligned} U &= u_0 + z u_1 + z^3 u_2 \\ V &= v_0 + z v_1 + z^3 v_2 \\ W &= w_0 + z w_1 + z^2 w_2 \end{aligned} \tag{1}$$

The kinematic hypothesis (1) can be generally written, using the UF, as

$$\begin{aligned} U &= F_0 u^{(0)} + F_1 u^{(1)} + F_2 u^{(2)} \\ V &= F_0 v^{(0)} + F_1 v^{(1)} + F_2 v^{(2)} \\ W &= F_0 w^{(0)} + F_1 w^{(1)} + F_2 w^{(2)} \end{aligned} \tag{2}$$

where F_τ, for τ = 0, 1, 2, are the thickness functions. In particular, for the in-plane displacements $(F_0, F_1, F_2) = (1, z, z^3)$ and for the out-of-plane displacement $(F_0, F_1, F_2) = (1, z, z^2)$. In conclusion, the displacement field (2) can be written in compact matrix form using the following recursive formula

$$\mathbf{U} = \sum_{\tau=0}^{2} \mathbf{F}_\tau \mathbf{u}^{(\tau)} \tag{3}$$

where $\mathbf{U} = [\,U(x,y,z,t) \quad V(x,y,z,t) \quad W(x,y,z,t)\,]^T$ is the three dimensional displacement vector and $\mathbf{u}^{(\tau)} = [\,u^{(\tau)}(x,y,t) \quad v^{(\tau)}(x,y,t) \quad w^{(\tau)}(x,y,t)\,]^T$ is the τth order generalized displacement component vector of the middle surface points $(z = 0)$. $\mathbf{F}_\tau$ is a 3×3 matrix, that is

$$\mathbf{F}_\tau = \begin{bmatrix} F_\tau & 0 & 0 \\ 0 & F_\tau & 0 \\ 0 & 0 & F_\tau \end{bmatrix} \tag{4}$$

This corresponds to a refined, higher-order shear deformation theory, as initially proposed by Kant [6]. For a laminate with n orthotropic layers, the lower and upper layer surfaces are defined by z_k and z_{k+1}, respectively, as illustrated in Fig. 1 for a 3-layered laminate. Since a laminated plate is taken into account, it should be also mentioned that the total plate thickness h is given by the following sum

$$h = \sum_{k=1}^{l} h_k \tag{5}$$

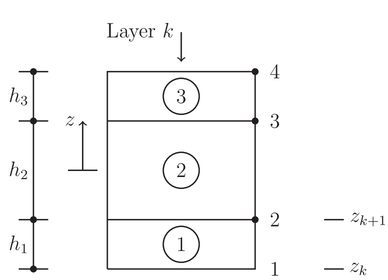

Fig. 1. A 3-layer laminate.

where $h_k = z_{k+1} - z_k$ is the generic thickness of the kth lamina and l is the total number of layers.

2.2. Deformation components

The generalized strain component vector of the τth order following the UF approach can be written as

$$\boldsymbol{\varepsilon}^{(\tau)} = \mathbf{D}_\Omega \mathbf{u}^{(\tau)}, \quad \text{for} \quad \tau = 0, 1, 2 \tag{6}$$

where $\boldsymbol{\varepsilon}^{(\tau)} = [\,\varepsilon_x^{(\tau)} \quad \varepsilon_y^{(\tau)} \quad \gamma_x^{(\tau)} \quad \gamma_y^{(\tau)} \quad \gamma_{xz}^{(\tau)} \quad \gamma_{yz}^{(\tau)} \quad \omega_{xz}^{(\tau)} \quad \omega_{yz}^{(\tau)} \quad \varepsilon_z^{(\tau)}\,]^T$ and $\mathbf{D}_\Omega$ is the kinematic partial differential operator

$$\mathbf{D}_\Omega = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 & 0 & 1 \end{bmatrix}^T \tag{7}$$

2.3. Stress components

The constituent material of the given plate layers is linearly elastic, thus, the constitutive equations in terms of stresses are defined lamina per lamina. The stress components are given by the Hooke law [13]

$$\boldsymbol{\sigma}^{(k)} = \overline{\mathbf{C}}^{(k)} \boldsymbol{\varepsilon}^{(k)} \tag{8}$$

where the stress component vector means $\boldsymbol{\sigma}^{(k)} = [\,\sigma_x^{(k)} \quad \sigma_y^{(k)} \quad \tau_{xy}^{(k)} \quad \tau_{xz}^{(k)} \quad \tau_{yz}^{(k)} \quad \sigma_z^{(k)}\,]^T$, the strain component vector is indicated as $\boldsymbol{\varepsilon}^{(k)} = [\,\varepsilon_x^{(k)} \quad \varepsilon_y^{(k)} \quad \gamma_{xy}^{(k)} \quad \gamma_{xz}^{(k)} \quad \gamma_{yz}^{(k)} \quad \varepsilon_z^{(k)}\,]^T$ and $\overline{\mathbf{C}}^{(k)}$ is the constitutive matrix for the kth lamina. For the sake of completeness the matrix at hand is reported below

$$\overline{\mathbf{C}}^{(k)} = \begin{bmatrix} \overline{C}_{11}^{(k)} & \overline{C}_{12}^{(k)} & \overline{C}_{16}^{(k)} & 0 & 0 & \overline{C}_{13}^{(k)} \\ \overline{C}_{12}^{(k)} & \overline{C}_{22}^{(k)} & \overline{C}_{26}^{(k)} & 0 & 0 & \overline{C}_{23}^{(k)} \\ \overline{C}_{16}^{(k)} & \overline{C}_{26}^{(k)} & \overline{C}_{66}^{(k)} & 0 & 0 & \overline{C}_{36}^{(k)} \\ 0 & 0 & 0 & \overline{C}_{44}^{(k)} & \overline{C}_{45}^{(k)} & 0 \\ 0 & 0 & 0 & \overline{C}_{45}^{(k)} & \overline{C}_{55}^{(k)} & 0 \\ \overline{C}_{13}^{(k)} & \overline{C}_{23}^{(k)} & \overline{C}_{36}^{(k)} & 0 & 0 & \overline{C}_{33}^{(k)} \end{bmatrix} \tag{9}$$

In Eq. (9) $\overline{C}_{nm}^{(k)}$, for $n, m = 1, 2, \ldots, 6$, represent the material constants in the Cartesian reference system, after the equations of transformation [13] are applied. By integrating the stress components through the thickness of the plate, the stress resultants are obtained

$$\mathbf{S}^{(\tau)} = \sum_{s=0}^{2} \mathbf{A}^{(\tau s)} \boldsymbol{\varepsilon}^{(s)}, \quad \text{for} \quad \tau = 0, 1, 2 \tag{10}$$

where $\mathbf{S}^{(\tau)} = [\,N_x^{(\tau)} \quad N_y^{(\tau)} \quad N_{xy}^{(\tau)} \quad N_{yx}^{(\tau)} \quad T_x^{(\tau)} \quad T_y^{(\tau)} \quad P_x^{(\tau)} \quad P_y^{(\tau)} \quad S_z^{(\tau)}\,]^T$ and $\mathbf{A}^{(\tau s)}$ are the stiffness constants [39] that can be evaluated as

$$\begin{aligned} A_{nm}^{(\tau s)} &= \sum_{k=1}^{l} \int_{z_k}^{z_{k+1}} \overline{C}_{nm}^{(k)} F_s F_\tau dz \\ A_{nm}^{(\tilde{\tau} s)} &= \sum_{k=1}^{l} \int_{z_k}^{z_{k+1}} \overline{C}_{nm}^{(k)} F_s \frac{\partial F_\tau}{\partial z} dz \quad \text{for} \quad \tau, s = 0, 1, 2 \\ A_{nm}^{(\tau \tilde{s})} &= \sum_{k=1}^{l} \int_{z_k}^{z_{k+1}} \overline{C}_{nm}^{(k)} \frac{\partial F_s}{\partial z} F_\tau dz \quad \text{for} \quad n, m = 1, 2, \ldots, 6 \\ A_{nm}^{(\tilde{\tau} \tilde{s})} &= \sum_{k=1}^{l} \int_{z_k}^{z_{k+1}} \overline{C}_{nm}^{(k)} \frac{\partial F_s}{\partial z} \frac{\partial F_\tau}{\partial z} dz \end{aligned} \tag{11}$$

In Eq. (11) the indices τ, s are related to the chosen thickness functions F_τ, F_s. When these indices are indicated as $\tilde{\tau}, \tilde{s}$, it means that the corresponding thickness functions are derived with respect to z, that is $\partial F_\tau / \partial z, \partial F_s / \partial z$. The subscripts n, m of Eq. (11) follow the relationships shown in Eq. (11) itself.