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A mathematical model for the behavior of laminated uniformly curved glass beams

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ABSTRACT

Laminated curved glasses as widely used elements in buildings urge to develop a mathematical model for their analysis and safer design. Large deflection theory is necessary in order to predict the true behavior of a laminated curved glass beam consisting of several glass layers bonded by soft interlayer PVB (Poly-Vinyl Butyral). In the present study, a mathematical model is developed for the analysis of a laminated circular arch or a laminated uniformly curved glass beam which is the special case of the laminated curved glass beams. Thus, three nonlinear, coupled partial differential equations governing the true behavior are derived in polar coordinates by applying variational and energy principles. Results of this model are compared with the results from the experiments and finite element model, and all of them are presented in figures to explain the true behavior.

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1. Introduction

Laminated glass units are widely used as architectural components of buildings. The units consist of two or more glass layers which are connected by an elastomeric polymer layer. They have some advantages such as filtering the solar radiation and eliminating the unwanted temperature effects. Among its most important advantages is related to safety or safety requirements set for the modern buildings to protect people from dangers due to broken glasses since the interlayer (PVB) between glass layers holds the fragments of glass together.

It has been quite a long time since laminated glasses are in use. First of all laminated glasses with flat shapes were used as their production was easy, and design parameters were available. Nowadays, the use of curved laminated glasses in modern buildings is increasing since it is possible to mold them into various bending shapes as byproducts.

Curved laminated glass beams have not been employed much in practice due to lack of information on their structural behavior. In general a curved beam differs from a straight beam due to its initial curvature. Most of the studies are about the linear rather than the nonlinear behavior of curved beams because of the mathematical complexity of the latter. In addition, laminated glasses easily undergo large deflections in transverse direction when subjected to lateral loads since thicknesses of glass layers used are very small compared to other dimensions. Therefore, it is necessary to develop a mathematical model based on large deflection theory to predict the strength and behavior of curved beams.

2. Previous research

The first study about laminated glass beams was conducted by Hooper [1]. He derived a mathematical model for the bending of laminated glass beams under four-point loading. The relevant differential equation in terms of applied bending moment and the axial force in one of the plies were solved by using Laplace transform. Hooper plotted three influence factors K_1, K_2 and K_3: proportional to the axial force in one of the plies, shear strain in the interlayer and central deflection, respectively. He noted that shear modulus of PVB can be written as a function of time approaching zero as it increases, since PVB is a viscoelastic material. Hooper carried out experiments on laminated glass beams under short (<3 min) and long (80 days) loading durations.

Behr et al. [2] conducted a series of experiments on layered, monolithic and laminated glass units, to verify the theoretical model for a laterally loaded, thin plates developed by Vallabhan [3]. They observed that stresses in layered glass units compared

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Nomenclature

$\underline{A}$	coefficient matrix
A_1, A_2	cross sectional areas of top and bottom glass plies
b	width of beam
E	modulus of elasticity of glass
G	shear modulus of interlayer
h	thickness of single glass ply
h_1, h_2	thickness of top and bottom glass plies
r_1, r_2	radius of top and bottom glass plies
r_I	radius of interlayer
I_1, I_2	moment of inertia of top and bottom glass arches
N_1, N_2	cross sectional forces at the top and bottom glass plies
num	number of discrete points along the beam
P	point load applied at middle of the beam
q	uniformly distributed load applied over the length of beam
$\underline{R}$	right hand side vector
s	arc length measured on the centroidal axis
t	thickness of the interlayer
U_m^i	membrane strain energy for the top and bottom plies
U_b^i	bending strain energy for the top and bottom plies
U_I	shear strain energy for the interlayer
u_1, u_2	displacements for the top and bottom plies in the θ direction
V_1	volume of top glass ply
V_2	volume of bottom glass ply
V_I	volume of interlayer
Ω	potential energy of applied loads
$\underline{w}$	lateral displacement vector
$w_{\max}$	maximum displacement in the plate
$wo(i)$	lateral displacement calculated in the previous step
θ, r	polar coordinates
ε_m^i	axial strain energy for the top and bottom plies
ε_b^i	bending strain energy for the top and bottom plies
γ_I	shear strain in the interlayer
α	under-relaxation parameter for convergence
Π	total potential energy of the system

to monolithic glass plates were larger near the corner and smaller near the center. It was also observed that at room temperature and below, maximum principal stresses near the corner of a laminated unit were slightly smaller than the theoretically predicted stresses at the same location in a monolithic glass plate. It can be said that at room temperature laminated glass unit behaved like a monolithic glass plate, whereas at higher temperatures it behaved like a layered glass unit. Therefore, the behavior of laminated glass unit was bounded by these two limiting cases.

Behr et al. [4] conducted experiments to consider the structural behavior of laminated glass under lateral pressure. Theoretical and experimental research was undertaken to investigate load–deflection behavior of monolithic, layered and laminated glass units under lateral pressure. Similarities between laminated and monolithic glass structural behavior were observed at room temperature and below, but the behavior of laminated glass changed towards layered glass behavior at elevated temperatures.

Edel [5] conducted three point bending experiments to investigate the temperature transition of laminated glass. Edel compared the results of his experiments with the results of the finite element model he developed. According to the results of the experiments the behavior of laminated glass approached to that of the monolithic model at temperatures much below the transition temperature of the PVB based interlayer, and approached to that of the layered model at temperatures in excess of the transition temperature.

Dawe [6] used finite element model to solve a circular arch with radius R and thickness t. The strain energy of the system was written in terms of tangential and normal components of displacements. To obtain the differential equations which govern the behavior of the arch, the first variation of the energy was used. By solving the governing differential equations, tangential and normal displacements were obtained. As a result Dawe improved the independently interpolated model by increasing the order of assumed displacement from cubic to quintic.

Rajasekaran and Padmanabhan [7] derived the governing equations for curved beams by employing large displacement theory. They used three dimensional small strain large displacement relations in cylindrical coordinates by ignoring the nonlinear terms associated with the displacement in z direction (w), since it was much smaller compared to the displacements in x and y directions. They derived the equilibrium equations and boundary conditions by using the principle of virtual displacements through by summing up the virtual work done by internal forces as the integral of the product of Kirchhoff stress tensor and virtual Green's strain tensor and the virtual work done by external forces.

Kang and Yoo [8] presented a consistent formulation for thin walled curved beams. They derived equilibrium equations predicting linear, large displacement and buckling behavior using the principle of minimum potential energy. To obtain the governing differential equations total potential energy of the system was written as the summation of strain energy and force potential. They neglected the nonlinear terms and considered the linear behavior of a curved beam. The governing coupled differential equations of the displacements and the boundary conditions for the curved beam were obtained by first variation of total potential energy. In their derivation the curvature effect was included. Analytical solution to the coupled equations was not easy: in order to solve the equations an approximation based on the binomial series was adopted by ignoring the higher order terms.

Lin and Heish [9] developed a closed form analytical solution for in plane laminated curved beam with variable curvatures. Prior to their study there were only a few papers devoted to laminated composite materials. Most of the studies were about the isotropic beam. To analyze curved beams numerically and analytically approximate methods were applied to the displacement field by the previous research. Lin and Heish [9] obtained the set of equations which were the general solutions of axial force, shear force, moment and rotation angle and displacement field for laminated curved beam in terms of the angle of tangential slope. They defined the arc length as a function of tangential slope.

Nonlinear behavior of the curved laminated glass beams are complex and requires iterative solution. The convergence problems even in the nonlinear solution of the curved beams are faced and investigated by several researchers [6,8–11].

Vallabhan et al. [12] developed a nonlinear model for two plates placed without an interlayer (i.e. layered glass plates) to determine the stresses and the limits of the behavior for the laminated glass units.

In 1993 Vallabhan et al. [13] improved their mathematical model developed for the layered glass plates given in Vallabhan et al. [12] to introduce a new mathematical model for the nonlinear stress analysis of the laminated glass plates by using the principle of minimum potential energy and variational calculus. Nonlinear

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