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Self-consistent simulation of the space charge dominated beams in an elliptical solenoid magnet

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ABSTRACT

The behaviour of a space charge dominated beam has been studied through an elliptical solenoid which is used for making a planar beam as well as for matching an axisymmetric beam to a system needing unequal beam sizes in the two transverse planes. We have first developed an envelope model based on the canonical description of the motion and derived ten independent first order differential equations for the beam sigma matrix elements by assuming canonically transformed Kapchinskij-Vladimirskij (KV) like distribution of the beam. In order to investigate the nonlinear space charge effect a 2D particle-in-cell method has been used. Five types of phase space distributions for the initial beam have been utilized to study the evolutions of envelope and emittance as a function of beam current for different initial beam conditions as the beam passes through the elliptical solenoid. It is shown that the evolution of beam sizes depends weakly on the form of initial distributions whereas the projected emittances in the two transverse planes strongly depend on the initial beam conditions and type of distributions.

1. Introduction

There is a continuous interest in the understanding of the space charge effects on focusing and transfer characteristics of intense charged particle beams in the low energy beam transport systems [1]. For focusing and transporting high intensity beams in the low energy section of accelerators several types of magnetic elements such as dipoles, quadrupoles, solenoids etc. are used [2,3]. Another type of magnetic element which is sometimes used to transform a circular high intensity beam to a planar configuration is the elliptical solenoid [4,5]. In a solenoid with circular cross-section, a beam is influenced by the uniform axial magnetic field and two hard-edge fields at both ends. These fields provide simultaneous focusing in both transverse planes and at the same time produce a rotation of the beam around the axis. In the case of an elliptical solenoid the elliptic apertures at both ends generate transverse field components which result in unequal focusing forces in the two transverse directions. The second order focusing effect of an elliptical solenoid looks like a skew quadrupole focusing; however the later is a first order effect. The asymmetric focusing characteristic of elliptical solenoid can be utilized to transport of high current elliptical sheet electron beam for high power microwave devices [6]. It can also be used in transverse matching of a high current axisymmetric beam to the system such as spiral inflector in a cyclotron, requiring unequal beam sizes and different orientation of phase ellipses in the two transverse planes [7]. The main advantage of an elliptical solenoid is that it provides continuous and unequal focusing in both transverse planes and requires a short axial length.

In a recent work we have investigated the beam optical properties of an elliptical solenoid magnet for intense beam using recursive sigma matrix method [8,9], and studied the envelope behaviour and emittance evolution in the usual laboratory frame projections that results from the coupling between the two transverse planes as a function of beam current and the coupling parameter. Since the analysis presented in the above work is based on the uniform distribution, results so obtained are applicable only in the cases where the space charge forces are linear.

In most of the high intensity accelerator facilities, the beam current transported from the ion source is of the order of miliamperes with energy ranges from 20 keV–120 keV and the density distribution of the beam is generally non-uniform. The nonlinear space charge field of the beam arising due to the non-uniform beam distribution leads to the emittance growth and is a serious concern particularly where the energy of the beam is low. This results in the appearance of small fraction of particles outside of the beam core and ultimately leads to the loss of beam particles. It is therefore, necessary and of practical importance to study the behavior of the space charge dominated beam more deeply through the elliptical solenoid magnet including the effect of nonlinear space charge forces.

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The analytical study of beam dynamics including the nonlinear space charge effects through the elliptical solenoid magnet is very complicated and thus it is necessary to use self-consistent numerical analysis for correct prediction of the beam behaviour. There are several advanced numerical tools such as particle-in-cell (PIC) simulation methods [10–12], Vlasov method [13], δf simulation method [14,15] etc. which are mainly used to realize the basic physics of the intense beams self-consistently. Among them PIC simulation is the most widely used numerical method for the investigation of the dynamics of charged particle beams.

At the Variable Energy Cyclotron Centre we are developing a 10 MeV, 5 mA compact proton cyclotron [16,17]. The beam transport line consists of a 2.45 GHz microwave ion source at an extraction voltage of 80 kV and two solenoid magnets to transport and match the proton beam to the entrance of the spiral inflector. Simulation results indicate that for better beam transmission and less emittance growth due to coupling between the two transverse planes in the spiral inflector, one needs convergent phase ellipses with different orientations in x and y planes and a comparatively smaller width in the y plane [7]. Thus to match an axisymmetric beam from the ion source to the entrance of the spiral inflector we require an elliptical solenoid between the second solenoid and the inflector. The purpose of the present paper is first to investigate the behaviour of magnetic field profile of the elliptical solenoid with ellipticity of the aperture and then to study the evolution of space charge dominated beam through it in more detail considering the nonlinear self-field effects.

In this paper, we first present a canonical description of the beam dynamics by constructing a Hamiltonian for transverse motions of a particle in the combined linear self-field and applied magnetic field of the elliptical solenoid. The Hamiltonian is then used to obtain the paraxial ray equations of motion in canonical form and to derive a set of envelope equations in the laboratory coordinate system for uniform density beam. Two conserved emittances [18,19], analogous to transverse emittances for the uncoupled system, are also derived for the transverse motion in an elliptical solenoid. To investigate the evolution of beam in the presence of nonlinear space charge field caused by the non-uniform particle distributions, we have developed a self-consistent 2D PIC code. A brief description of the steps involved in the development of 2D PIC code is presented. In order to perform the detailed numerical simulation of beam dynamics through the elliptical solenoid we have first modeled a small magnet and studied its parameters in detail by computing the magnetic field using a 3D code. The computed field is then used in the envelope equations and in PIC code to study the focusing properties of the magnet. We have used five types of phase space distributions for the initial beam and studied the evolutions of envelope and emittance as a function of beam current for different initial beam conditions as the beam passes through the elliptical solenoid.

2. Beam envelope model

In this section, we present a canonical description of the motion of a single particle in the presence of linear self and applied fields and then derive the coupled differential equations of the beam sigma matrix elements to obtain the expressions for beam envelope and emittance. We consider an intense continuous beam of particles of charge q and rest mass m propagating through an elliptical solenoid with an average axial momentum $P = m\gamma\beta c$, where β and γ are the usual relativistic parameters and c is the speed of light in vacuum. In the laboratory frame, we use a right handed Cartesian coordinate system x, y and z with unit vectors $\hat{x}, \hat{y}$ and $\hat{z}$ respectively. As it is customary in accelerator physics, we use $s = z$, the distance along the axial direction aligned with the beam axis and x, y as the transverse coordinates from the beam axis. In the present analysis it is assumed that the trajectories of the particles remain very close to the axis and transverse beam sizes are small compared to the radii of beam pipes, coils, etc. It is also assumed that the transverse velocities of beam particles are much smaller compared to the average axial velocity (i.e. $\dot{x}, \dot{y} \ll v = \beta c$).

2.1. Hamiltonian and equations of motion

Under the paraxial approximation of beam transport, the potential in the elliptical solenoid up to second order in the cylindrical coordinate system can be written as [8]

$$\Phi(r,\theta,s) = C(s) - C''(s)\left(\frac{r}{2}\right)^2 + D(s)\left(\frac{r}{2}\right)^2 \cos(2\theta) \tag{1}$$

where $C(s)$ and $D(s)$ are functions of the axial distance s and prime denotes the derivative of the function with respect to s. The first two terms in Eq. (1) are the usual terms used in the conventional solenoid. The third term arises from the asymmetric pole faces and is similar to the quadrupolar term. In terms of coordinates x, y and s, the potential Φ in Eq. (1), can be expressed as

$$\Phi(x,y,s) = C(s) - \frac{1}{4}(C''(s) - D(s))x^2 - \frac{1}{4}(C''(s) + D(s))y^2 \tag{2}$$

Near the beam axis and for small transverse excursions of the beam in the transport line, the leading order terms of the components of the magnetic field at any point (x,y,s) can be expressed as

$$B_x(x,y,s) = -\frac{1}{2}(B'_s(s) - D(s))\, x \tag{3a}$$

$$B_y(x,y,s) = -\frac{1}{2}(B'_s(s) + D(s))\, y \tag{3b}$$

$$B_s(x,y,s) = B_s(s) \tag{3c}$$

where $B_s(s) = C'(s)$ and $B'_s(s) = C''(s)$ are the field and its derivative on the axis of the solenoid. The function $D(s)$ is related to the field gradient along x and y directions and depends on the shape of elliptic cross-section of the solenoid. Thus the magnetic field of an elliptical solenoid consists of a longitudinal solenoidal field and a superimposed s dependent quadrupolar field which results in unequal focusing forces in the two transverse directions. The value of elliptic parameter $D(s)$ within the linear approximation is given by

$$D(s) = \left(\frac{B_x(R_x)}{R_x} - \frac{B_y(R_y)}{R_y}\right) \tag{4}$$

where R_x and R_y are the semi major and semi minor axes of the elliptic pole face of the solenoid and $B_x(R_x)$ and $B_y(R_y)$ are the magnetic fields at the tip of the pole face along x and y directions respectively.

In order to obtain the magnetic vector potential $\mathbf{A}$ in the elliptical solenoid, we choose the gauge

$$xA_x + yA_y = 0 \tag{5}$$

Using this gauge and introducing two functions $F(x,y,s)$ and $G(x,y,s)$, three components of the vector potential $\mathbf{A}$ can be expressed as

$$A_x = -yF,\ A_y = xF,\ A_s = G \tag{6}$$

The components of magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ become

$$B_x = \frac{\partial G}{\partial y} - x\frac{\partial F}{\partial s}, \quad B_y = -\frac{\partial G}{\partial x} - y\frac{\partial F}{\partial s} \text{ and } \quad B_s = x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} + 2F \tag{7}$$