Composites: Part B 56 (2014) 126–136

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Optimized sinusoidal higher order shear deformation theory for the analysis of functionally graded plates and shells

J.L. Mantari, C. Guedes Soares *

Centre for Marine Technology and Engineering (CENTEC), Instituto Superior Técnico, Technical University of Lisbon, Av. Rovisco Pais, 1049-001 Lisbon, Portugal

ARTICLE INFO

Article history:
Received 12 February 2013
Received in revised form 18 June 2013
Accepted 31 July 2013
Available online 9 August 2013

Keywords:
A: Laminates
B: Elasticity
C: Analytical modeling
Functionally graded materials

ABSTRACT

The optimization of the sinusoidal higher order shear deformation theory (HSDT) for the bending analysis of functionally graded shells is presented in this paper for the first time. The HSDT includes the stretching effect and their shear strain shape functions (sin(*mz*) and cos(*nz*)) contain the parameters "*m*" and "*n*" that need to be selected by providing displacements and stresses which produce close results to 3D elasticity solutions. The governing equations and boundary conditions are derived by employing the principle of virtual work. A Navier-type closed-form solution is obtained for functionally graded plates and shells subjected to transverse load for simply supported boundary conditions. Numerical results of the optimized sinusoidal HSDT are compared with the FSDT, other quasy-3D hybrid type HSDTs, reference solutions, and 3D solutions. The key conclusions that emerge from the present numerical results suggest that: (a) the optimization procedure is beneficial in terms of accuracy; and (b) it is possible to gain accuracy keeping the unknown's constant by performing the optimization procedure shown in this paper.

1. Introduction

Classical composite shells are extensively used in the industry but they always require the designer's effort to tailor different laminate's properties to suit a particular application. Some classical composite structures suffer from discontinuity of material properties at the interface of the layers and constituents. Therefore, the stress fields in these regions create interface problems. Furthermore, large plastic deformation of the interface may trigger the initiation and propagation of cracks in the material [1]. In order to alleviate this problem, functionally graded materials (FGM), which are classified as novel composite materials, were proposed by Bever and Duwez [2], and then developed and successfully used in industrial applications since 1984 [3].

Functionally graded structures made possible to graduate the material properties through the thickness and avoid abrupt changes in the stress and displacement distributions. Currently FGMs are alternative materials widely used in aerospace, nuclear reactor, energy sources, biomechanical, optical, civil, automotive, electronic, chemical, mechanical, and shipbuilding industries. FGMs are important due to outstanding properties of being able to resistant high temperature gradients, strong mechanical performance and reduce the possibility of catastrophic fracture. Therefore, FGMs performs very well in high temperature environment applications such as heat exchanger tubes, thermoelectric generators, furnace linings, electrically insulated metal ceramic joints. Recently, the application of FGMs can be seen in micro and nanodevices (it should be keep in mind that with nanotechnology one may be able to create new materials and devices with potential applications in medicine, electronics, biomaterials and energy production). Then, FGMs is a topic that needs considerable attention.

In general, FGMs are both macroscopically and microscopically heterogeneous advanced composites which are made for example from a mixture of ceramics and metals with continuous composition gradation from pure ceramic on one surface to full metal on the other. In fact, FGMs are materials with spatial variation of the material properties. However, in most of the applications available in the literature, as in the present work, the variation is through the thickness only. Therefore, it evidences the early state of development of functionally graded materials.

Recently, several researchers have provided results on functionally graded plates (FGPs) and shells. Both analytical and numerical solutions can be found in the literature. An interesting literature review also may be found in the paper of Birman and Byrd [4], see also Mantari and Guedes Soares [5–9]. An updated literature review of functionally graded materials can be found in the work by Jha et al. [10]. Therefore, for completeness, in the present article, only the relevant and related work on functionally graded shells performed during the last two years is described.

Carrera et al. [11] studied the static analysis of FG plates and shells. The stretching effect was included in the mathematical formulation and the importance of the transverse normal strain

* Corresponding author. Tel.: +351 218 417607; fax: +351 218 474015.
E-mail address: guedess@mar.ist.utl.pt (C. Guedes Soares).

1359-8368/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compositesb.2013.07.027

effects in the mechanical prediction of stresses of FG plates and shells was remarked. Recently, Neves et al. [12,13] and Ferreira et al. [14] presented a quasi-3D hybrid type (polynomial and trigonometric) shear deformation theory for the static and free vibration analysis of FGPs by using meshless numerical method. Their formulation can be seen as a generalization of the original Carreras's Unified Formulation (CUF), by introducing different non-polynomial displacement fields for in-plane displacements, and polynomial displacement field for the out-of-plane displacement. Mantari and Guedes Soares [5–9] presented bending results of FGM by using news non-polynomial HSDTs. In [7,8], the stretching effect was included and improved results of displacement and in plane normal stresses compared with [5,6] were found.

The well-known Carrera's Unified formulation was recently extended to include non-polynomial shear strain shape function in their formulation [12–14], and the need of new non-polynomial shear strain shape function, which can be adapted to this advanced generalized formulation for perhaps better performance, is demanding. In the present work, the optimization of the well-known sinusoidal shear deformation theory is presented for the first time. Therefore, a new, attractive, and very accurate shear strain shape function which potentially can be used in other numerical methods and analytical higher order shear deformation formulations is given.

In the present paper, the six-unknown well-known sinusoidal HSDT formulated for shells is optimized for the bending analysis of FGPs. The theory complies with the tangential stress-free boundary conditions on the plate boundary surface, and thus a shear correction factor is not required. The plate governing equations and their boundary conditions are derived by employing the principle of virtual work. A Navier-type analytical solution is obtained for shells subjected to transverse load for simply supported boundary conditions. Benchmark results for the displacement and stresses of functionally graded rectangular plates are obtained. The results of present optimized HSDTs are compared with 3D exact, quasy-exact, and with other closed-form solutions published in the literature. Finally, a new, simple, attractive and very accurate optimized sinusoidal shear deformation theory is presented for the first time.

2. Theoretical formulation

The rectangular doubly-curved shell made of FGM of uniform thickness, h, is shown in Fig. 1. The ξ_1 and ξ_2 curves are lines of curvature on the shell mid-surface, $\xi_3 = \zeta = 0$, while $\xi_3 = \zeta$ is a straight line normal to the mid-surface. The principal radii of normal curvature of the reference (middle) surface are denoted by R_1 and R_2. The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point $(\xi_1, \xi_2, \pm h/2)$ on the outer (top) and inner (bottom) surfaces of the shell, is given as follows:

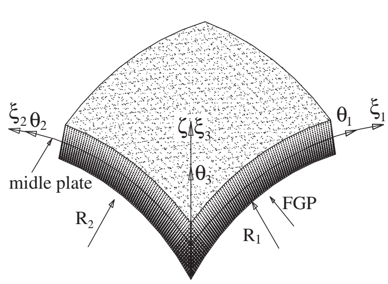

Fig. 1. Geometry of a functionally graded plate.

$$\bar{u} = \left(1 + \frac{\zeta}{R_1}\right)u + \zeta\left[y^*\theta_1 + q^*\frac{\partial\theta_3}{a_1\partial x} - \frac{\partial w}{a_1\partial x}\right] + m\sin(\zeta/m)\theta_1,$$
$$\bar{v} = \left(1 + \frac{\zeta}{R_2}\right)v + \zeta\left[y^*\theta_2 + q^*\frac{\partial\theta_3}{a_2\partial y} - \frac{\partial w}{a_2\partial y}\right] + m\sin(\zeta/m)\theta_2,$$
$$\bar{w} = w + \cos(\zeta/n)\theta_3, \tag{1a-c}$$

where $u(\xi_1,\xi_2)$, $v(\xi_1,\xi_2)$, $w(\xi_1,\xi_2)$, $\theta_1(\xi_1,\xi_2)$, $\theta_2(\xi_1,\xi_2)$ and $\theta_3(\xi_1,\xi_2)$ are the six unknown displacement functions of the middle surface of the panel, whilst $y^* = -\cos\left(\frac{h}{2m}\right)$ and $q^* = -\cos\left(\frac{h}{2n}\right)$ (being h the thickness of the shell), a_1 and a_2 are scalar values inherent to the type of shells. These scalar values are associated to the vectors tangent to the ξ_1 and ξ_2 coordinate lines, respectively, for more details readers may consult the interesting book written by Reddy [15].

As can be noticed in Eq. 1(a-c), the displacement field includes the parameters m and n into the shear strain shape functions. These parameters are selected in Section 4 with the idea to obtain close to 3D results.

In the derivation of the necessary equations, small elastic deformations are assumed, i.e. displacements and rotations are small, and obey Hooke's law. The starting point of the present thick shell theory is the 3D elasticity theory [15], expressed in general curvilinear (reference) surface-parallel coordinates; while the thickness coordinate is normal to the reference (middle) surface as given in Fig. 1. The strain–displacement relations, based on this formulation, are written as follows:

$$\varepsilon_1 = \frac{1}{A_1}\left(\frac{\partial\bar{u}}{\partial\xi_1} + \frac{1}{a_2}\frac{\partial a_1}{\partial\xi_2}\bar{v} + \frac{a_1}{R_1}\bar{w}\right),$$
$$\varepsilon_2 = \frac{1}{A_2}\left(\frac{\partial\bar{v}}{\partial\xi_2} + \frac{1}{a_1}\frac{\partial a_2}{\partial\xi_1}\bar{u} + \frac{a_2}{R_2}\bar{w}\right),$$
$$\varepsilon_3 = \frac{\partial\bar{w}}{\partial\xi_3},$$
$$\varepsilon_4 = \frac{1}{A_2}\frac{\partial\bar{w}}{\partial\xi_2} + A_2\frac{\partial}{\partial\xi_3}\left(\frac{\bar{v}}{A_2}\right),$$
$$\varepsilon_5 = \frac{1}{A_1}\frac{\partial\bar{w}}{\partial\xi_1} + A_1\frac{\partial}{\partial\xi_3}\left(\frac{\bar{u}}{A_1}\right),$$
$$\varepsilon_6 = \frac{A_2}{A_1}\frac{\partial}{\partial\xi_1}\left(\frac{\bar{v}}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial\xi_2}\left(\frac{\bar{u}}{A_1}\right), \tag{2a-f}$$

where

$$A_1 = \left(1 + \frac{\xi_3}{R_1}\right)a_1, \quad A_2 = \left(1 + \frac{\xi_3}{R_2}\right)a_2. \tag{3a-b}$$

and $\xi_i (i = 1, \ldots, 6)$ represent strain components; $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the displacements on the surface (ξ_1, ξ_2, ξ_3) and a_1 and a_2 the vectors tangent to the ξ_1 and ξ_2 coordinate lines.

Introduction of Eqs. (1a-c) into the relations given in Eqs. (2a-f) of a moderately shallow and deep shell supplies the following strain–displacement relations, valid for a doubly-curved panel under consideration:

$$\varepsilon_{xx} = \varepsilon^0_{xx} + \zeta\varepsilon^1_{xx} + m\sin(\zeta/m)\varepsilon^2_{xx},$$
$$\varepsilon_{yy} = \varepsilon^0_{yy} + \zeta\varepsilon^1_{yy} + m\sin(\zeta/m)\varepsilon^2_{yy},$$
$$\varepsilon_{zz} = -\frac{1}{n}\sin(\zeta/n)\varepsilon^5_{zz},$$
$$\varepsilon_{yz} = \varepsilon^0_{yz} + \cos(\zeta/n)\varepsilon^3_{yz} + \cos(\zeta/m)\varepsilon^4_{yz},$$
$$\varepsilon_{xz} = \varepsilon^0_{xz} + \cos(\zeta/n)\varepsilon^3_{xz} + \cos(\zeta/m)\varepsilon^4_{xz},$$
$$\varepsilon_{xy} = \varepsilon^0_{xy} + \zeta\varepsilon^1_{xy} + m\sin(\zeta/m)\varepsilon^2_{xy}. \tag{4a-f}$$