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The influence of the intermolecular surface forces on the static deflection and pull-in instability of the micro/nano cantilever gyroscopes

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ABSTRACT

In this paper, the effects of van der Waals and Casimir forces on the static deflection and pull-in instability of a micro/nano cantilever gyroscope with proof mass at its end are investigated. The micro/nano gyroscope is subjected to coupled bending motions which are related by base rotation and nonlinearities due to the geometry and the inertial terms. It is actuated and detected by capacitance plates which are placed on the proof mass. The extended Hamilton principle is used to find the equations governing the static behavior of the clamp-free micro/nano gyroscopes under electrostatic, Casimir and van der Waals forces. The equations of static motion are discritized by Galerkin's decomposition method. The nonlinear equilibrium equations are solved analytically using homotopy perturbation method (HPM). The static response of the micro/nano gyroscopes to variations in the DC voltage across the drive and sense electrodes is obtained and the effects of different parameters on pull-in instability are investigated. The presented results can be used for accurate estimations of the instability and performance of the micro/nano gyroscopes.

1. Introduction

The micro/nano mechanical systems (M/NEMS) have progressed in testing and fabricating new devices recently. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, make them even more attractive [1,2]. M/NEMS devices find variety of applications such as micropumps, micromirrors, microphones, microresonators, random access memory, nanotweezers for miniaturized robotics, super sensitive sensors, gyroscopes and devices for high-frequency operation and fast switching in communication networks [3]. Vibratory micro/nano gyroscopes use suspending mechanical parts to measure rotation. They have no gyratory component that require bearings, and for this reason they can be easily miniaturized and batch production using micromachining methods. They operate based on the energy interchange between two modes of vibration of a structure. Many investigations and designs have been developed to improve the efficiency of microgyroscopes [4–13]. Bhadbhade et al. [11] and Bhadbhade and Jalili [12] have studied coupling vibrations of a microcantilever gyroscope. The primary vibrations (drive oscillation) are produced in the beam using a piezoelectric actuator and secondary vibration are measured by piezoelectric sensors placed on the beam. They have derived equation of motion and analyzed relation between the frame rotation and gyroscope coupling. Esmaeili et al. [13] have modeled a suspended cantilever beam with a tip mass under general base excitation and electrostatic forces. The linear beam has been considered to vibrate in all the three directions, while subjected to a base rotational motion around its longitudinal direction. They have utilized the Euler–Bernoulli theory to represent the microbeam and a linear approximation to represent the electrostatic force. Ghomem et al. [9] have developed a mathematical model of linear microcantilever gyroscope with a proof mass at its end under electrostatically actuator and detector. The microgyroscope undergoes two flexural vibrations that are coupled by base rotation about the microbeam longitudinal axis. They have studied the static deflection of microgyroscope and the relationship between the base rotation and gyroscopic coupling.

Comparing to MEMS devices, the operation of NEMS devices is different because of the importance of the intermolecular surface forces such as van der Waals and Casimir forces which can be neglected at micrometer scale. Even in the absence of electrostatic actuation, when the gap between cantilever tube and the rigid substrate is very small the pull-in phenomenon can occur because of the intermolecular forces [14–16].

The intermolecular surface forces are especially significant when the nanosystems are working in vacuum without the effect of capillary forces and the separations between movable components are in the sub-micrometer range [17]. For separations much less than the plasma wavelength (for a metal) or much less than the absorption wavelength (for a dielectric) of the material

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constituting the surfaces (typically below 20 nm), the retardation, which is a result of the finite propagation speed of the electromagnetic field, is not significant [17]. In this case, the intermolecular force between two surfaces is simplified as the van der Waals attraction [18]. The Casimir force arises from the polarization of adjacent material bodies, separated by distances of less than a few microns. less than a few microns [19]. Van der Waals force and Casimir force can both be connected with the existence of zero-point vacuum oscillations of the electromagnetic field [20–22]. The microscopic approach to the modeling of both van der Waals and Casimir forces can be formulated in a unified way using Quantum Field Theory [17,20–23]. It is found that the Casimir force is generally effective at larger separation distances between the bodies than the van der Waals force. Whereas the Casimir force between semi-infinite parallel plates is inversely proportional to the fourth power of the gap, van der Waals force is inversely proportional to the third power of the gap. The dependence of these forces on the dielectric properties of the plates and the filling medium is studied in detail in Ref. [22]. It is important to note that van der Waals and Casimir forces cannot in general be considered to simultaneously act in MEMS, since they describe the same physical phenomenon at two different length scales.

Effect of van der Waals force on the pull in instability of electrostatically actuated rectangular microplates has been studied by Batra et al. [14,15]. Lin and Zhao [24] adopted a one degree of freedom mass spring model to study the influence of Casimir force on the nonlinear behavior of nanoscale electrostatic actuators. Dequesnes et al. [25] studied the pull-in voltage characteristics of several nanotube electromechanical switches, such as double-wall carbon nanotubes suspended over a graphitic ground electrode. They proposed parametrized continuum models for coupled electrostatic and van der Waals energy domains. They compared the accuracy of the continuum models with atomistic simulations. Their numerical simulations based on continuum models closely match the experimental data reported for carbon nanotube-based nanotweezers.

Ding et al. [17]. presented an analysis of Casimir effect with surface roughness, conductivity and temperature corrected on the deformation of a membrane strip structure. They provided a way of designing a membrane strip with high resistance to collapse. Ramezani et al. [19,26,27] investigated the two point boundary value problem of the deflection of nano-cantilever subjected to Casimir and electrostatic forces using analytical and numerical methods to obtain the instability point of the nanobeam. They computed the pull-in parameters of the beam under combined effects of electrostatic and Casimir forces. In their analytical approach, the nonlinear differential equation of the model was transformed into the integral form by using the Green's function of the cantilever beam. Then, closed-form solutions were obtained by assuming an appropriate shape function for the beam deflection to evaluate the integrals. They used the same method to investigate the influence of van der Waals force on the pull-in voltage and deflection of nanomechanical switches using a distributed parameter model. The fringing field effect was also taken into account in their model.

In this research, the static deflection and instability of the nonlinear clamp-free micro/nano beam gyroscopes are studied. The gyroscope consists of a beam with a rigid (proof) mass attached to it and undergoes coupled flexural–flexural motions. The system is actuated and detected by capacitive electrodes which are placed on the proof mass and is subjected to base rotation and intermolecular surface forces (Casimir and van der Waals forces). The equations of static motion are derived by extended Hamilton principles and the method of homotopy perturbation and numerical method are used to find the static deflection of the system. The input intermolecular surface and electrostatic forces have an upper limit beyond which the restoring force of the micro/nano structure can no longer resist them, and consequently the structure spontaneously collapses. This behavior is known as pull-in instability, and the upper limit of input voltage is called pull-in voltage [28–30]. Studying on the pull-in phenomenon is important in the design process of micro/nano gyroscopes to determine the sensitivity, instability and the dynamic behavior of devices. The static response and pull-in instability of the micro/nano gyroscope to variations in the intermolecular forces across the drive and sense electrodes are obtained and the effects of different parameters on pull-in behavior are investigated.

2. Modeling and formulation

Fig. 1 shows a micro/nano beam gyroscopes with a proof mass attached to its end under base rotation, electrostatic and intermolecular surface forces. The gyroscope undergoes two coupled bending motions which are coupled by rotation and nonlinear large curvature. The deformation of a beam is depicted by means of the longitudinal displacement $u(s,t)$, the transverse displacements $v(s,t)$ and $w(s,t)$ along the y and z axis respectively, and the torsional angle $\phi(s,t)$.

The curvature vector of the deformed beam can be obtained as [29].

$$\rho = \rho_\xi e_\xi + \rho_\eta e_\eta + \rho_\zeta e_\zeta \quad (1)$$

where

$$\begin{aligned} \rho_\xi &= (\phi' - \psi' \sin\theta) \\ \rho_\eta &= (\psi' \cos\theta \sin\phi + \theta' \cos\phi) \\ \rho_\zeta &= (\psi' \cos\theta \cos\phi - \theta' \sin\phi) \end{aligned} \quad (2)$$

where the prime stands for $\partial/\partial s$.

Here xyz is a global orthogonal coordinate system, while $\xi\eta\zeta$ is local coordinate system.

For microcantilever, the beam's neutral axis to be inextensional; that is, $e = 0$. The inextensionality constraint equation thus is [30]

$$1 - (1 + u')^2 - v'^2 - w'^2 = 0 \quad (3)$$

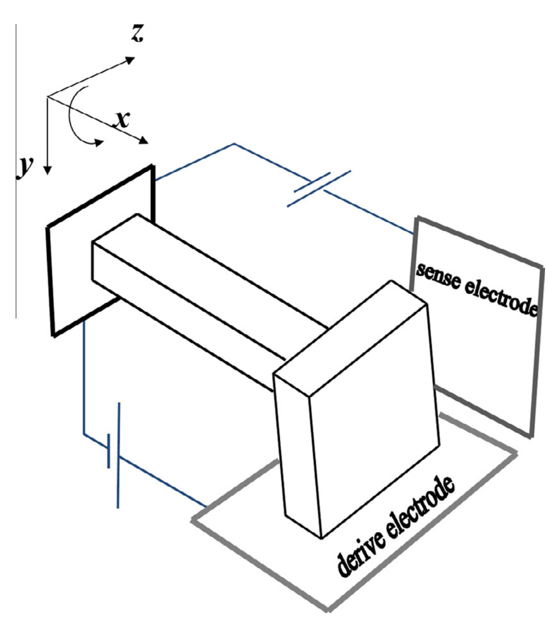

Fig. 1. Cantilever beam with proof mass under electrostatic actuation and detection.