Nuclear Instruments and Methods in Physics Research A 724 (2013) 20–26

Contents lists available at SciVerse ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

# Isochronicity correction in the CR storage ring

S. Litvinov [a], D. Toprek [b], H. Weick [a,*], A. Dolinskii [a]

[a] GSI, Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany
[b] VINCA Institute of Nuclear Sciences, Belgrade University, P.O Box 522, 11000 Belgrade, Serbia

ARTICLE INFO

*Article history:*
Received 6 March 2013
Received in revised form
7 May 2013
Accepted 15 May 2013
Available online 22 May 2013

*Keywords:*
CR
Achromat
Storage ring
Nuclear masses

ABSTRACT

A challenge for nuclear physics is to measure masses of exotic nuclei up to the limits of nuclear existence which are characterized by low production cross-sections and short half-lives. The large acceptance Collector Ring (CR) [1] at FAIR [2] tuned in the isochronous ion-optical mode offers unique possibilities for measuring short-lived and very exotic nuclides. However, in a ring designed for maximal acceptance, many factors limit the resolution. One point is a limit in time resolution inversely proportional to the transverse emittance. But most of the time aberrations can be corrected and others become small for large number of turns. We show the relations of the time correction to the corresponding transverse focusing and that the main correction for large emittance corresponds directly to the chromaticity correction for transverse focusing of the beam. With the help of Monte-Carlo simulations for the full acceptance we demonstrate how to correct the revolution times so that in principle resolutions of $\Delta m/m = 10^{-6}$ can be achieved. In these calculations the influence of magnet inhomogeneities and extended fringe fields are considered and a calibration scheme also for ions with different mass-to-charge ratio is presented.



## 1. Introduction

The Collector Ring of the FAIR facility [2] is a symmetric, achromatic ring with two arcs, two straight sections and a total circumference of 221.5 m. It is designed for operation at a maximum magnetic rigidity of $B\rho = 13$Tm [1]. It will be operated in three ion-optical modes, two of them providing fast pre-cooling of either antiprotons or radioactive ion beams [1]. In the third mode (isochronous optics) the CR will be operated as a Time-of-Flight (ToF) spectrometer for the mass measurement of exotic very short-lived nuclei ($T_{1/2} > 20\,\mu$s) produced and selected in flight with the Super-FRS fragment separator [3]. This technique for mass measurements has been developed at the ESR at GSI [4] and meanwhile was also used successfully in the CSRe at IMS Lanzhou [5] and the RIKEN Rare-RI ring is under construction [6].

The first basic condition for isochronicity is the well known equation [4] which describes the revolution time ($T$) of ions stored in the ring and its deviation ($\Delta T$) depending on the difference in mass-to-charge ratio ($m/q$) or different velocity ($v$)

$$\frac{\Delta T}{T} = \frac{1}{\gamma_t^2} \cdot \frac{\Delta(m/q)}{(m/q)} + \left(\frac{\gamma^2}{\gamma_t^2} - 1\right)\frac{\Delta v}{v} + \frac{\Delta T_{others}}{T} \qquad (1)$$

where $\gamma$ is the relativistic Lorentz factor and $\gamma_t$ is the transition energy of the ring. The isochronous condition is reached when $\gamma = \gamma_t$. This means, the second term in Eq. (1) vanishes and $\Delta(m/q)$ can be determined from the observed $\Delta T$. It is remarkable that $T$ is directly proportional to $m/q$. But in addition to the intended time differences for different masses and the important first order isochronicity condition the resolution depends on more additional influences on time-of-flight which are collected in $\Delta T_{others}$. These aberrations can also have higher order contributions. Effects of nonlinear field errors, fringe fields of magnets, closed orbit distortion and the transverse emittance negatively act on the resolution [7].

The general layout of the CR has been described in Ref. [8]. The isochronous mode has been calculated for different values of $\gamma_t$ [9]. In this paper we will always refer to the mode for $\gamma_t = 1.67$ which allows to measure masses up to $m/q = 3.1\ u/e$. In this case the momentum acceptance becomes $\Delta p/p = \pm 0.5\%$ when having at the same time a transverse acceptance of 100 mm mrad in both planes.

A correction of time-of-flight aberrations due to a large transverse phase space distribution is essential or otherwise the goal of large phase acceptance in the CR cannot be achieved with sufficient time resolution [7]. The simple reason for this influence is that ions which oscillate around the optical axis have a longer path length for one turn which leads to a corresponding time spread. As it was indicated before [10] and will be also shown in this article these time-of-flight aberrations are directly related to

* Corresponding author. Tel.: +49 615 971 21 58.
*E-mail address:* h.weick@gsi.de (H. Weick).

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http://dx.doi.org/10.1016/j.nima.2013.05.057

the chromatic aberrations of the optical system. It is known that in a straight system with only magnetic lenses defined by magnets outside of the beamline a correction is not possible [11]. With additional electrostatic elements this would be possible [12], however, these are not available for the high rigidity of the CR beam. At least a correction in a system with a bent optical axis is possible [12]. A recipe for such a full second or higher order achromat was given by Brown [13], and partially this was also applied in the spectrometer TOFI [14] for time-of-flight mass measurements of nuclear masses. However, when a storage ring is constructed in this way a four fold symmetry system which is imaging violates the basic stability criterion of a nonzero phase advance for a closed ring [15].

This is also the result of a general approach to higher order achromats by Wan and Berz [16], the minimum symmetry needed for a second order achromat is a four-fold symmetry, but based on the Lie algebra description of the system [17] it is found that the tunes of the individual cells should be integer. A good approximation to a second order achromat may be found with the method of extending a full achromat by straight matching sections for adjusting the tunes. In this case the overall second order geometric aberrations can be kept zero as pure quadrupoles are free of them and by an overcorrection with the sextupoles at least the tune shift be corrected (so-called pseudo achromat) [18]. As the matching section can be short compared to the whole arc section this usually leads to systems close to a full achromat.

This dilemma may not be so crucial in a ring for only few revolutions but for commissioning and other detection techniques like Schottky pickups stability of motion must be guaranteed. In this paper we will show a solution that does not lead to full achromaticity over one turn but to a correction in the limit of many turns. By averaging the time of flight over many turns still a good resolution for a large phase space can be obtained. This less stringent requirement also helps to avoid the usual problem of large higher order field components, which in turn could impose other higher order aberrations.

In the CR we want to use ToF detectors inside the ring which do measure the revolution time for each turn. This means that we must observe the ions over a series of turns and see how large the remaining error will be. This is anyway necessary as the resolution of the detector itself is limited and only averaging over a larger number of turns helps to obtain the desired resolution.

## 2. Calculation of second-order time deviations

The relative revolution time difference between an arbitrary and the reference particle can be expressed in terms of the initial coordinates as a Taylor series in a second-order approximation [19,20]:

$$\begin{aligned}\frac{\Delta T}{T} = \frac{T-T_0}{T_0} &= (t|x)_c x + (t|a)_c a + (t|\delta)_c \delta + (t|xx)_c x^2 \\ &+ (t|xa)_c xa + (t|aa)_c a^2 + (t|yy)_c y^2 + (t|yb)_c yb \\ &+ (t|bb)_c b^2 + (t|x\delta)_c x\delta + (t|a\delta)_c a\delta + (t|\delta\delta)_c \delta^2 \end{aligned} \quad (2)$$

where $(x)$, $(y)$ are the transverse position coordinates and $(a)$, $(b)$ the transverse momenta divided by the forward momentum of a reference particle ($p_0$). The coefficients of the Taylor series correspond to the partial derivatives of the first coordinate before the line with respect to the following. The index $c$ marks coefficients normalized by the total time-of-flight $nT_0$, where $n$ is the number of turns and $T_0$ is the revolution time of the reference particle. The fractional momentum deviation $\delta$ is given by $p = p_0(1+\delta)$.

In the first-order achromatic ring, with $(x|\delta) = (a|\delta) = 0$, the first-order transverse time coefficients $(t|x)$ and $(t|a)$ vanish simultaneously [10]. The necessary condition to be an isochronous ring in first order is $(t|\delta) = 0$, i.e. $\gamma = \gamma_t$. The second-order isochronous condition is fulfilled when $(t|\delta\delta) = 0$, which can be corrected with one family of sextupole magnets installed in a dispersive section of the ring.

With such a good adjustment of $\gamma_t$ even to higher order in an achromatic ring the largest contribution to $\Delta T$ comes from the second-order geometric time aberrations. For a beam of one species in an ideal isochronous ring without higher order field errors or closed orbit distortions only pure betatron motion exists. In such a ring the time spread is directly related to the transverse emittances ($\varepsilon_{x,y}$) [7]

$$\left(\frac{\Delta T}{T}\right)_{emitt.} \approx \frac{1}{4}(\varepsilon_x \langle \gamma_x \rangle + \varepsilon_y \langle \gamma_y \rangle) \quad (3)$$

where $\varepsilon_{x,y}$ is the transverse beam emittance, which can be regarded as the ring acceptance, and $\langle \gamma_{x,y} \rangle$ are the Twiss parameters $\gamma_{x,y}$ averaged over the whole circumference of the ring. This result simply describes the longer path length of ions traveling with larger betatron amplitude over one turn.

From the maximum deviation given in Eqs. (1) and (3) one can derive the mass resolution over the whole beam emittance. For the CR, with acceptance 100 mm mrad in both planes, the limit of mass resolution would be about $10^{-5}$, which is insufficient for precise mass measurements. Therefore, in order to reach the necessary resolution of $10^{-6}$ the transverse emittance would have to be limited to 10 mm mrad in both planes. As a result, the transmission of the ions into the ring would be reduced drastically. However, the mass resolving power can be improved using second-order corrections and keeping the transverse emittance large.

For the second order terms it is useful to investigate their behavior over many revolutions. As the system is bound and the betatron oscillations have a non-zero phase advance the initial coordinates of a single ion change from turn to turn within fixed limits. The time deviations over many turns simply sum up. The fluctuations are periodic with the tune and have an average value of zero. Therefore, the time deviations given by a single matrix coefficients which depend on mixed initial coordinates will oscillate with a fixed maximum amplitude. When looking at the relative time difference over many turns these contributions will become small. The quadratic terms however will continue to increase from turn to turn and the relative time deviation converges towards a constant value. Fig. 1 illustrates this behavior.

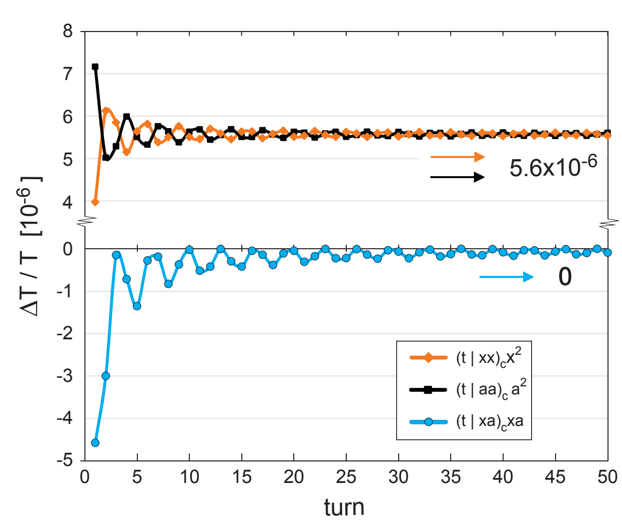


**Fig. 1.** Evolution of the relative second-order geometric time aberrations as a function of the number of turns in the CR for a single ion with start positions $x$ and $a$ on the edge of the acceptance of 100 mm mrad. The inserted numbers show the limit over many turns.