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Transient analysis of FGM and laminated composite structures using a refined 8-node ANS shell element

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ABSTRACT

In this paper, we investigate the vibration analysis of functionally graded material (FGM) and laminated composite structures, using a refined 8-node shell element that allows for the effects of transverse shear deformation and rotary inertia. The properties of FGM vary continuously through the thickness direction according to the volume fraction of constituents defined by sigmoid function, but in this method, their Poisson's ratios of the FGM plates and shells are assumed to be constant. The finite element, based on a first-order shear deformation theory, is further improved by the combined use of assumed natural strains and different sets of collocation points for interpolation the different strain components. We analyze the influence of the shell element with the various location and number of enhanced membrane and shear interpolation. Using the assumed natural strain method with proper interpolation functions the present shell element generates neither membrane nor shear locking behavior even when full integration is used in the formulation. The natural frequencies of plates and shells are presented, and the forced vibration analysis of FGM and laminated composite plates and shells subjected to arbitrary loading is carried out. In order to overcome membrane and shear locking phenomena, the assumed natural strain method is used. To validate and compare the finite element numerical solutions, the reference solutions of plates based on the Navier's method, the series solutions of sigmoid FGM (S-FGM) plates are obtained. Results of the present theory show good agreement with the reference solutions. In addition the effect of damping is investigated on the forced vibration analysis of FGM plates and shells.

1. Introduction

Whether they are used in civil, marine or aerospace, most structures are subjected to dynamic loads during their operation. Therefore, there exists a need for assessing the natural frequency and transient response of structures. The vibration of thin shells was discussed in the work of Love [1]. Since then many researchers have dealt with shell vibration using classical thin-shell theory. In particular, Donnell [2] used the classical thin shallow-shell theory to understand the free vibration behavior of shells and subsequent research has been studied by Leissa [3], Qatu [4] and Liew [5]. The work on the free vibration analysis of isotropic plates and shells using a shell element can be founded in Lee and Han [6]. Ascione et al. [7] and Fraternali et al. [8–10] studied composite curved beams and shells. Recent work on the vibration analysis and transient response of plates and shells can be founded in Park et al. [11], Lee and Han [12] and Han et al. [13].

The Reissner–Mindlin assumptions have been used in the improvement of such elements and consequently the mass matrix includes rotary inertia effects. The accuracy of the results is improved by rotary inertial effect in the mass matrix and transverse shear effects in the stiffness matrix. However, there are serious defects such as locking phenomena. As commonly accepted, two kinds of locking phenomena may occur in curved shear flexible bending element, namely shear locking and membrane locking. While the shear locking may possibly occur in both flat and curved shear flexible bending element, the membrane locking occurs only in curved thin shell. Bathe and Dvorkin [14] proposed an eight-node shell element, named as MITC8, that avoids membrane and shear locking. The strain tensor was expressed in terms of the covariant components and contravariant base vectors. The performance of this element was quite satisfactory and suggested the promising results in very complex shell structures. Bucalem and Bathe [15] have improved in earlier publications the MITC8 shell elements [14] and concluded that while it performed quite effectively in some cases, in a few analyses the element presented a very stiff behavior rendering it not useful and improvements are still desirable. Kim and Park [16] and Kim et al. [17] presented an 8-node shell finite

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element. In 8-node shell element, the persistence of locking problems was found to continue through numerical experiments on the standard test problem of MacNeal and Harder [18].

Recently, Han et al. [19] studied the new combination of sampling points for the assumed natural strain and concluded that while it performed quite effectively in some cases, in a few analyses of very thin-walled structures the element presented less accurate results.

Functionally graded material (FGM) is a special kind of composites in which the material properties vary smoothly and continuously from one surface to the other. These materials are microscopically inhomogeneous and are typically made from isotropic components. One of the main advantages of FGM is that it mitigates severe stress concentrations and singularities at intersections between interfaces usually presented in laminate composites due to their abrupt transitions in material compositions and properties. If a high external pressure is applied to the composite plate and shell structures, the high stresses occurred in the structure will affect its integrity, and the structure, as the result, susceptible to failure. For these reasons, understanding the mechanical behavior of FGM plates and shells are very important to assess the safety of the shell and plate structure. Chung and Chi [20] propose a sigmoid FGM, which is composed of two power-law functions to define a new volume fraction. Chi and Chung [21] indicate that the use of a sigmoid FGM can significantly reduce the stress intensity factors of a cracked body. Recent work on the vibration, buckling and geometrically non-linear analysis of FGM plates and shells can be founded in Sofiyev and Schnack [22], Han et al. [23,24], Li and Wang [25], Bich et al. [26–30] and Dung and Hoa [31].

The aim of this paper is to propose an improvement of the most useful curved quadrilateral shell finite element, which is clearly, from a practical point of view, the 8-node element. In order to improve the 8-node ANS shell element, a new combination of sampling points and shaper functions are adopted for the vibration analysis of FGM and laminated composite structures. To validate the present shell element models, the numerical examples are investigated and compared with those solutions from the previous literatures. The solutions of the free and forced vibration analysis are numerically illustrated in a number of figures to show the influence of the types of dynamic loads, the damping effect and the loading time effect in FGM and laminated composite structures.

2. Improvement of shell element

2.1. Kinematics of shell

The geometry of an 8-noded shell element with six degrees of freedom is shown in Fig. 1.

Kinematic equations for the first-order shear deformation theory including extension of the normal line can be obtained from the 3D equations of the theory of elasticity by using a well-known first-order approximation of a vector function with respect to the coordinate ξ_3. (Rikards et al. [32]). Further well-known geometric relations for the shell in normal coordinates are used.

Let us assume that vector $\overline{\mathbf{P}}$ characterizes the position of an arbitrary point of the shell in the initial reference state (see point B in Fig. 2) and vector $\overline{\mathbf{Q}}$ is the position of the same point (point B') in the deformed state. The position of a point at the midsurface of the shell (point A) in the initial state is characterized by vector $\mathbf{P}$, and position of the same point in the deformed state (point A') is characterized by vector $\mathbf{Q}$. Normal curvilinear coordinates $\xi^i = [\xi^\alpha, \xi_3]$ at the midsurface of the shell in the initial state are defined by the right-handed triad of the base vectors $[\mathbf{a}_\alpha, \mathbf{a_3}]$. Here, a unit vector $\mathbf{a_3}$ is normal to the midsurface of shell. Therefore (see Fig. 2).

$$\overline{\mathbf{P}}(\xi^{\mathbf{i}}) = \mathbf{P}(\xi^\alpha) + \xi_3\mathbf{a_3} \tag{1}$$

Similarly, curvilinear coordinates $\bar{\xi}^i = [\bar{\xi}^\alpha, \bar{\xi}_3]$ in the deformed state is defined by the triad of vectors $[\mathbf{A}_\alpha, \mathbf{A_3}]$. In the deformed state, the vector $\mathbf{A_3}$ may be not perpendicular to the midsurface of shell.

Vector function $\overline{\mathbf{Q}}$ can be expanded in a Taylor's series with respect to coordinate ξ_3 normal to the midsurface of shell.

$$\overline{\mathbf{Q}}(\xi^{\mathbf{i}}) = \mathbf{Q}(\xi^\alpha) + \xi_3\nabla\otimes\mathbf{Q}\cdot\mathbf{a_3} + \cdots \tag{2}$$

Here $\nabla\otimes\mathbf{Q} = \mathbf{A}_\alpha\otimes\mathbf{a}^\alpha = \mathbf{G}$ is a second-order tensor (see Luri'e, [33]), which characterizes the gradient of strains with respect to coordinate ξ_3, ∇ is a Hamiltonian operator, and $\otimes$ is a dyadic product of the tensors.

Further, the notations for displacement at the midsurface $\mathbf{u}$ and displacement of an arbitrary point of shell $\bar{\mathbf{u}}$ are introduced. The following expressions can be written (see Fig. 2).

$$\overline{\mathbf{Q}} = \overline{\mathbf{P}} + \bar{\mathbf{u}}, \quad \mathbf{Q} = \mathbf{P} + \mathbf{u} \tag{3}$$

From expressions (1)–(3) the representation of the displacement $\bar{\mathbf{u}}$ of an arbitrary point of the shell for the first-order approximation can be obtained

$$\bar{\mathbf{u}}(\xi^\alpha) = \mathbf{u}(\xi^\alpha) + \xi_3\varphi(\xi^\alpha) \tag{4}$$

Here $\boldsymbol{\varphi}$ is vector of rotation at the midsurface of shell

$$\varphi(\xi^\alpha) = \mathbf{A}_3(\xi^\alpha) - \mathbf{a}_3(\xi^\alpha) \tag{5}$$

where

$$\mathbf{A}_3 = \mathbf{G}\cdot\mathbf{a}_3 \tag{6}$$

A three-dimensional Green's strain tensor in the linear case (infinitesimal strain theory) is given by

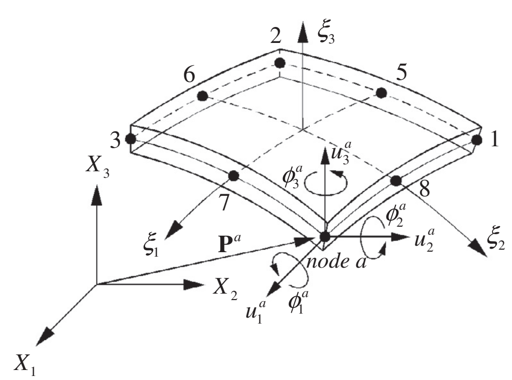

Fig. 1. Geometry of 8-node shell element with six degrees of freedom.

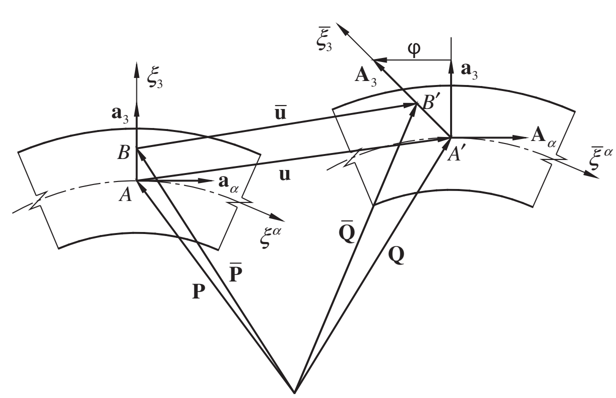

Fig. 2. Kinematics of the first-order shear deformation theory.