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Simplified two-fluid current–voltage relation for superconductor transition-edge sensors

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A B S T R A C T

We propose a simplified current–voltage (IV) relation for the analysis and simulation of superconductor transition-edge sensor (TES) circuits. Compared to the conventional approach based on the effective TES resistance, our expression describes the device behavior more thoroughly covering the superconducting, transitional, and normal-state for TES currents in both directions. We show how to use our IV relation to perform small-signal analysis and derive the device's temperature and current sensitivities based on its physical parameters. We further demonstrate that we can use our IV relation to greatly simplify TES device modeling and make SPICE simulation of TES circuits easily accessible. We present some interesting results as examples of valuable simulations enabled by our IV relation.

1. Introduction

After about two decades' development, the superconductor transition-edge sensor technology has become mature and reliable enough to allow the deployment of mid-scale TES detector arrays with tens to hundreds of pixels in state-of-the-art scientific instruments [1,2]. In contrast, the simulation and design techniques for TES circuits remain less advanced, relying primarily on the small-signal model [3] developed in the early age of TES research. The small-signal model is very important in revealing the small-signal circuit response and stability conditions. However, it also has its limitations in supporting circuit analysis and simulation. The temperature and current sensitivity α and β lie in the heart of the small-signal formalism, yet the small-signal model itself cannot relate these vital characteristics to the device's physical parameters. Because of this, simulation model based on the small-signal formalism must contain not only the device's physical parameters but also α and β which in principle should be determined by the physical parameters. Simulations based on such model cannot help in selecting the device's physical parameters to optimize the circuit performance and meet the design requirement [4]. Also, being a small-signal model in nature, it cannot support studies that are not limited to small signals, notably DC analysis which is indispensable for determining the circuit's bias values and working conditions, and analysis of AC-biased TES circuits which experience large current and voltage swings. What is more, semi-manual analysis based on the analytical solution of the small-signal model fails to take advantage of the extremely powerful, robust, and efficient computer-based electronic design automation (EDA) tools for large-scale integrated circuit design that have been developed in the past few decades. Such tools can not only dramatically enhance design efficiency and reduce human error but help to gain deep insight into the system behavior, and thus they have become indispensable in modern electronic circuit design and research.

As the scale and complexity of the TES circuits grows [5], it is our belief that TES circuit design and simulation will increasingly benefit from leveraging the power of proven practices in modern integrated circuit design and relying on sophisticated EDA tools that are already available. For doing so, it is necessary to develop accurate and reliable TES device models that can be integrated in modern circuit simulators. In Ref. [6], we reported the modeling techniques for building a two-fluid TES device model. Our method was based on the conceptually simple and elegant approach of using polynomial controlled sources only, and the resulting device model can be used in many circuit simulators. However, it has the disadvantage of being complicated because some clever and unusual techniques must be used. This makes the model and simulations based on it less accessible, especially to non-experts in device modeling.

In this work, we try to overcome the limitations of the small-signal model by suggesting a two-fluid IV relation for the TES that

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is more generally applicable. By using the IV relation to perform analysis and simulation of TES circuits, we derive how the temperature and current sensitivities of the TES depend on its physical parameters. We also show how we can dramatically simplify device modeling for SPICE simulation by taking advantage of advanced behavioral modeling features available in modern circuit simulators. To demonstrate the validity of the IV relation and its usefulness in supporting TES circuit simulation and research, we present some interesting results and discuss their implications for circuit operation. For the readers' benefit, we include all device model and SPICE simulation files in the appendix. They are tested and directly usable. Thanks to the drastic simplification in our device modeling, the resulting model and simulation files are easily accessible to a general audience not specialized in device modeling. Therefore, our techniques and results can be readily adopted by a wide community of TES researchers to help to modernize TES circuit simulation and design.

2. IV relation based on the two-fluid theory

2.1. Limitation of the effective-resistance approach

Accurate modeling of the IV characteristic of the TES is essential for correctly understanding and describing its behavior. In our previous work [6], we followed the conventional method to model the TES as an effective resistance. Assuming a two-fluid physical model and considering both the supercurrent and normal current, we can express the effective TES resistance as

$$R_{tes} = \frac{V_{tes}}{I_{c0}\left(1-\frac{T}{T_c}\right)^{3/2} + V_{tes}/(C_R R_n)}, \tag{1}$$

where V_{tes} is the voltage across the TES, T and T_c are the temperature and critical temperature of the TES, I_{c0} is the 0-temperature critical current, R_n is the normal-state resistance, and C_R is the ratio of the TES normal current resistance in the transition regime to R_n. The dependence of the critical current I_c on the temperature has been modeled by the simple BCS relation

$$I_c = I_{c0}(1-T/T_c)^{3/2}. \tag{2}$$

The effective resistance in Eq. (1) correctly models the TES in the transition regime when the TES current is greater than the critical current (and thus the total TES current consists of both a supercurrent and a normal current). This is the working regime for a DC-biased TES under small-signal perturbation. In this case, a small-signal analysis based on the sensitivity of R_{tes} with respect to the TES temperature and current, α and β, works very well. However, if we are interested in studying the TES in a wider range of working conditions, Eq. (1) becomes inadequate. For instance, in AC-biased TES circuits (see Section 4.2.4), the TES current experiences large swings in both directions, and the TES enters and exits the superconducting state in each period of the oscillation (see Fig. 7(b)). Such AC-biased TES circuits cannot be studied by perturbative expansion of the effective resistance in Eq. (1) around some constant DC value. As one more example, in high-energy radiation (such as X and γ ray) detectors, the large input signal can saturate the TES and drive it to the normal-state where Eq. (1) does not apply (since it is only valid for $T < T_c$). Aside from these fundamental issues, there are also practical problems in using Eq. (1) to build a generic device model for TES. If we assign a positive value to the 0-temperature critical current I_{c0}, then Eq. (1) can only describe the IV relation of the TES in one direction of the current flow, which causes difficulty for simulating AC-biased TES with alternating current. Also, in our circuit simulation work [6], we found that modeling the TES using Eq. (1) can sometimes cause the circuit simulator to give a false superconducting solution of the circuit (notice $R_{tes} = 0$ and $V_{tes} = 0$ always satisfy Eq. (1) because of its form, even for $T > T_c$).

2.2. Expressing the IV relation with step functions

In order to build a device model that can be used to analyze and simulate TES circuits under all possible working conditions of interest, we consider different combination of the TES temperature and current and model the behavior of the TES under each condition separately. We can divide our discussion into the following:

- The TES temperature is greater than the critical temperature, i.e., $T > T_c$. In this case, the TES can be modeled as a resistance R_n, and its IV relation is given by $V_{tes} = IR_n$, for TES current I in both directions (i.e. both positive and negative I).
- The TES temperature is below the critical temperature, i.e., $T < T_c$. In this case, a supercurrent whose magnitude does not exceed the critical current I_c (positive by definition) given in Eq. (2) can flow through the TES. The TES voltage V_{tes} depends on both the magnitude and direction of the TES current I. Specifically,
  - $I > 0$, two possibilities.
    - $I < I_c$. All current is supercurrent, and $V_{tes} = 0$.
    - $I > I_c$. I_c of the total current is supercurrent, the rest is normal current. $V_{tes} = (I - I_c)C_R R_n$.
  - $I < 0$, also two possibilities.
    - $I > -I_c$, or $|I| < I_c$. All current is supercurrent, and $V_{tes} = 0$.
    - $I < -I_c$, or $|I| > I_c$. $-I_c$ of the total current is supercurrent, the rest is normal current. $V_{tes} = (I + I_c)C_R R_n$.

Summarizing all the cases discussed, we can express the IV relation of the TES as

$$\begin{aligned} V_{tes} = {} & \theta(T-T_c)IR_n \\ & +\theta(T_c-T)\theta(I-I_c)(I-I_c)C_R R_n \\ & +\theta(T_c-T)\theta(-I-I_c)(I+I_c)C_R R_n, \end{aligned} \tag{3}$$

where θ is the step function, and the temperature-dependent $I_c(T)$ is given by Eq. (2). We can rewrite Eq. (3) in the following slightly more concise form:

$$\begin{aligned} V_{tes} = {} & \theta(T_c-T)\theta\left(abs(I)-I_{c0}\left(1-\frac{T}{T_c}\right)^{3/2}\right)C_R R_n \\ & \cdot\left(I-sgn(I)I_{c0}\left(1-\frac{T}{T_c}\right)^{3/2}\right) + \theta(T-T_c)R_n I, \end{aligned} \tag{4}$$

where abs and sgn are the absolute-value function and sign function respectively.

The IV relation in Eqs. (3) and (4) looks unfamiliar and cumbersome. However, as is evident from its derivation, it covers all working conditions of the TES and incorporates the effective resistance relation in Eq. (1) as a special case. As we will demonstrate in Section 4, it allows us to easily build robust device models suitable for accurate and reliable SPICE-based simulations of TES circuits. Perhaps surprisingly, it is also convenient for theoretical analysis of the TES circuits (see Section 3), in spite of its intimidating appearance.

2.3. Thermal equation written with the IV relation

The thermal process must be considered for a complete description of the TES physics. Using Eq. (3) or Eq. (4) for the IV relation of the TES, we can express the Joule power in the TES as IV_{tes}. Thus, if we model the device as an absorber-TES-substrate