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Shear buckling of orthotropic rectangular graphene sheet embedded in an elastic medium in thermal environment

M. Mohammadi [a,b,*], A. Farajpour [c], A. Moradi [a], M. Ghayour [b]

[a] Department of Engineering, Ahvaz Branch, Islamic Azad University, Ahvaz, Iran
[b] Department of Mechanical Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran
[c] Young Researches and Elites Club, North Tehran Branch, Islamic Azad University, Tehran, Iran

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This article is dedicated to Professor Hossien Mohammadi (Amir Hossien Mohammadi) on the occasion of his 60th birthday

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ABSTRACT

In this study, the buckling behavior of orthotropic rectangular nanoplate is studied. Nonlocal elasticity theory has been implemented to investigation the shear buckling of orthotropic single-layered graphene sheets (SLGSs) in thermal environment. Using the principle of virtual work, the governing equations are derived for the orthotropic rectangular nanoplates. Differential quadrature method (DQM) is employed and numerical solutions for the critical shear buckling load are obtained. Six boundary conditions are investigated. The influence of surrounding elastic medium, temperature change, material properties and effect of boundary conditions on the shear buckling analysis of orthotropic SLGSs is studied. Numerical results show that the critical shear buckling load of SLGSs is strongly dependent on the small scale coefficient.

1. Introduction

Due to the rapid development of technology, especially in micro- and nano-scale fields, one must consider small scale effects and atomic forces to obtain solutions with acceptable accuracy. Neglecting these effects in some cases may result in completely incorrect solutions and hence wrong designs. As conduction of experiments in nano-level are difficult to control and theoretical atomistic models are computationally intensive for relatively large scale nanostructures, the continuum and semi-continuum [1,2] models have been proven to be important tools in the study of the nanostructures. There are various micro-continuum theories such as couple stress theory [3], micro-morphic theory [4], strain gradient elasticity theory [5] and nonlocal elasticity theory [6]. Among these theories, nonlocal elasticity theory has been widely applied to various problems of physics including lattice dispersion of elastic waves and dislocation mechanics [7]. In nonlocal elasticity theory, the small-scale effects are captured by assuming the stress at a point as a function not only of the strain at that point but also a function of the strains at all other points in the domain. A lot of work has already been done on the continuum models for stability analysis of CNTs or similar micro or nanobeam like elements [8,9]. Recently, Murmu and Pradhan [10] applied nonlocal Timoshenko theory and DQM for the stability analysis of embedded single-walled carbon nanotubes. Behfar and Naghdabadi [11] investigated the vibration behavior of multi-layer graphene sheets (MLGSs) embedded in an elastic medium. Sakhee-pour et al. [12] investigated the behavior of SLGSs using molecular structural mechanics. Moosavi et al. [13] investigated vibration analysis of nanorings using nonlocal continuum mechanics and shear deformable ring theory. In their article, they showed that the nonlocal effects play an important role in the vibration of nanorings and cannot be neglected. Mohammadi et al. [14] studied the free transverse vibration analysis of circular and annular graphene sheets with various boundary conditions using the nonlocal continuum plate model. They are obtained explicit relations for natural frequencies through direct separation of variables. They applied new version of differential quadrature method for vibration analysis of embedded single layer circular nanoplate. The applications of graphene sheets in electro-mechanical resonators [15], mass sensors and atomistic dust detectors [16] are recently reported. Because of these applications, the increasing level of knowledge of buckling behavior of graphene sheets becomes important for engineering design and manufacture. Eltaher et al. [17] exploited the nonlocal functional grade material for static and stability analysis of nanobeams. In that paper, their results addressing the significance of the material distribution profile, nonlocal effect, and boundary conditions on the bending and buckling behavior of nanobeams are presented.

* Corresponding author at: Department of Engineering, Ahvaz Branch, Islamic Azad University, Ahvaz, Iran. Tel.: +98 9369712728; fax: +98 311 3912628.
E-mail address: m.mohamadi@me.iut.ac.ir (M. Mohammadi).

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A version of nonlocal elasticity was proposed by Eltaher et al. [18] to formulate a nonlocal version of Euler–Bernoulli beam theory. They assumed that the material properties of FG nanobeams are assumed to vary through the thickness according to a power law. Aksencer and Aydogdu [19] reported vibration and buckling of nanoplates with nonlocal elasticity theory. In that paper Navier solution and levy type solution are used. In their paper, they studied buckling of nanoplate under biaxial in-plane pre-load. They showed that nonlocality effects should be considered for nanoscale plates. Farajpour et al. [20] investigated effect of surface and small scale parameter on the axisymmetric buckling of circular graphene sheet on thermal environment. They valid their results with molecular dynamic (MD) and showed that their results were exactly in agreement with the results of MD. Microtubules (MTs) are protein organized in a network that is interconnected with micro-filaments and intermediate filaments to form the cytoskeleton structures Microtubules. The mechanical properties of MTs play an important role in process such as cell division and intracellular transport. There have been a number of mathematical studies in recent years dealing with the mechanical properties of MTs [21–24]. Bending analysis of microtubules using nonlocal Euler–Bernoulli beam theory were used by Civalek and Demir [22] who consider nonlocal elasticity for bending analysis of microtubules protein. They discussed about the influence of nonlocal parameter on the static response of microtubules protein. Akgöz and Civalek [23] applied strain gradiant theory for buckling analysis of protein microtubules. In that paper, the governing equations for buckling and related boundary conditions are obtained by using the variational principle in conjunctions with the strain gradient elasticity. The strain gradient elasticity and modified couple stress were used for buckling analysis of axially loaded micro-scaled beams by Akgöz and Civalek [24]. They showed that the critical buckling load predicted by the modified strain gradient elasticity theory (MSGT) is about 3.2 times than that predicted by the modified couple stress theory (MCST) for micro-scaled beam. Ghorbanpour Arani et al. [25] studied the thermal effect on buckling analysis of a double-walled carbon nanotube embedded on the Pasternak foundation. In their paper, the interaction between matrix and the outer tube is modeled as a Pasternak foundation. In that paper, the interaction between matrix and the outer tube is modeled as a Pasternak foundation and the results are obtained of numerical simulation indicate that for any specific circumferential wave number (n), the nonlocal critical buckling pressure is related directly to the axial half wave number (m). Hashemi and Samaei [26] have used nonlocal elasticity model to investigate the buckling analysis of micro-scale plates. Their results showed that buckling loads of biaxially compressed micro-scale plate depend on the nonlocal parameter. Narendar [27] presented buckling analysis of micro and nano-scale plates based nonlocal scale effects. In that paper, he used two-variable refined plate theory for buckling of nanoplate under biaxial in-plane pre-load. He showed that the refined plate theory did not require shear correction factor. In his paper, he did not consider effect of elastic medium on the buckling analysis of rectangular nanoplate. Mahmoud et al. [28] investigated static analysis of nanobeams including surface effects and nonlocal elasticity theory. In that paper, the informations about the forces between atoms, and the internal length scale are proposed by the nonlocal Eringen model. Farajpour et al. [29] studied axisymmetric buckling of the circular graphene sheets with the nonlocal continuum plate model. In that paper, the buckling behavior of circular nanoplates under uniform radial compression is studied. Explicit expressions for the buckling loads are obtained for clamped and simply supported boundary conditions. It is shown that nonlocal effects play an important role in the buckling of circular nanoplates. Nateghi and Salamat-talab [30] investigated Thermal effect on size dependent behavior of functionally graded microbeams based on modified couple stress theory. They used modified couple stress in their work. They showed that Study of power index of material distribution proved that the behavior of FG microbeams differ considerably from homogeneous isotropic ones.

To the best knowledge of authors, it is the first time the nonlocal elasticity theory has been successfully applied to analysis of shear buckling for the orthotropic rectangular nanoplate.

In the current work attempt is made to investigate the shear buckling of rectangular graphene sheets embedded in an elastic medium under thermal environment. The influence of the surrounding elastic medium on the critical shear buckling load of the SLGSs is investigated. Both Winkler-type and Pasternak-type models are employed to simulate the interaction of the graphene sheets with a surrounding elastic medium. Differential quadrature method (DQM) is being used for the numerical solutions of the associated governing differential equations. The obtained results are subsequently compared with valid result reported in the literature. The effects of (a) small scale parameter, (b) stiffness of the surrounding elastic medium, (c) aspect ratio, (d) boundary conditions, (e) Thermal effect and (f) material properties on the critical shear buckling load of SLGSs are examined. The present work would be helpful while designing NEMS/MEMS devices using graphene sheets.

2. Nonlocal plate model

Nonlocal continuum theory states that the stress at a reference point x in an elastic continuum depends not only on strain at x but also on the strains at all other points x' in the body [4,6]. The basic equations for a linear homogenous elastic body using nonlocal elasticity theory are

$$\sigma_{ij} + f_i = \rho \ddot{u}_i \tag{1}$$

$$\sigma_{ij}(x) = \int \lambda(|x - x'|, \mu) C_{ijkl} \varepsilon_{kl}(x') dV(x'), \quad \forall x \in V, \tag{2}$$

where σ_{ij}, f, ρ and u_i are the nonlocal elasticity stress tensor, mass density, body forces, and the displacement vector at point x, respectively. C_{ijkl} is the local stress tensor at any point x' in the body which is related to the strain tensor ε_{kl}. $\lambda\,(|x - x'|, \eta)$, $|x - x'|$ and $\mu = (e_0 l_i/a)$ are the nonlocal kernel function, Euclidean distance, and material constant that depends on the internal characteristic length l_i (such as the C–C bond length, lattice parameter) and external characteristic length a (like graphene sheet length, wave length, crack length), respectively. The parameter e_0 is Eringen's nonlocal elasticity constant suitable to each material. The differential form of Eq. (1) can be written as [20]:

$$(1 - (e_0 l_i)^2 \nabla^2)\sigma^{nl} = [C][\{\varepsilon\} - \{\lambda\}\Delta T] \tag{3}$$

where ":" represents the double dot product. ∇^2 is the Laplacian operator and is given by $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$. In two-dimensional forms the stress–strain relations are written as

$$\begin{Bmatrix} \sigma_{xx}^{nl} \\ \sigma_{yy}^{nl} \\ \sigma_{xy}^{nl} \end{Bmatrix} - (e_0 l_i)^2 \nabla^2 \begin{Bmatrix} \sigma_{xx}^{nl} \\ \sigma_{yy}^{nl} \\ \sigma_{xy}^{nl} \end{Bmatrix} = \begin{bmatrix} E_1/(1-\upsilon_{12}\upsilon_{21}) & \upsilon_{12}E_2/(1-\upsilon_{12}\upsilon_{21}) & 0 \\ \upsilon_{12}E_2/(1-\upsilon_{12}\upsilon_{21}) & E_2/(1-\upsilon_{12}\upsilon_{21}) & 0 \\ 0 & 0 & 2G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} - \alpha_{xx}\Delta T \\ \varepsilon_{yy} - \alpha_{yy}\Delta T \\ \varepsilon_{xy} \end{Bmatrix} \tag{4}$$

where E_1 and E_2 are Young's modulus, G_{12} is shear modulus, and υ_{12}, υ_{21} are Poisson's ratios and α_{xx} and α_{yy} are the coefficient of thermal expansion along the principle material directions x and y, respectively. σ_{xx}^{nl}, σ_{yy}^{nl} and σ_{xy}^{nl} represent the nonlocal stress tensors. The strains in terms of displacement components in the middle surface can be written