Composites: Part B 56 (2014) 638–646

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Determination of material parameters for discrete damage mechanics analysis of carbon-epoxy laminates

Ever J. Barbero [a,*], F.A. Cosso [b]

[a] Cátedra de Excelencia, Universidad Carlos III de Madrid, Spain
[b] GRA, West Virginia University, USA

ARTICLE INFO

Article history:
Received 22 May 2013
Received in revised form 21 July 2013
Accepted 26 August 2013
Available online 7 September 2013

Keywords:
A. Polymer-matrix composites (PMCs)
B. Transverse cracking
C. Damage mechanics
C. Finite element analysis (FEA)
Parameter identification

ABSTRACT

Discrete damage mechanics (DDM) refers to micromechanics of damage constitutive models that, when incorporated into commercial finite element software via user material subroutines, are able to predict intralaminar transverse and shear damage initiation and evolution in terms of the fracture toughness of the composite. A methodology for determination of the fracture toughness is presented, based on fitting DDM model results to available experimental data. The applicability of the DDM model is studied by comparison to available experimental data for Carbon Epoxy laminates. Sensitivity of the DDM model to h- and p-refinement is studied. Also, prediction of modulus vs. applied strain is contrasted with ply discount results and the effect of in situ correction of strength is highlighted.

1. Introduction

Prediction of damage initiation and accumulation in polymer matrix, laminated composites is of great interest for the design, production, certification, and monitoring of an increasingly large variety of structures. Matrix cracking due to transverse tensile and shear deformations is considered in this manuscript. Matrix cracking is normally the first mode of damage and, if left unmitigated often leads to other modes such as delamination, and even fiber failure of adjacent laminas due to load redistribution. Furthermore, extensive cracking leads to increased permeability and exposes the fibers to deleterious environmental attack.

The earliest, simplest and least accurate modeling technique to address matrix damage is perhaps the ply discount method [1, Section 7.3.1]. Ply discount is used in this work as a baseline for contrasting predictions obtained with the Discrete Damage Mechanics (DDM) method. Although many other models exist, such as [2–12], Abaqus PDA [13–15], and several plugins [16,17], this manuscript focuses on DDM because its inherent features make it attractive.

Briefly, DDM [18] is a constitutive modeler that is inherently mesh independent, thus not requiring the user to choose a *characteristic length*. Furthermore, only two material parameters, the fracture toughness in modes I and II, are required to predict both initiation and evolution of transverse and shear damage. Since transverse and shear strengths are not used to predict damage initiation, but rather fracture toughness is used, DDM automatically accounts for in situ effects. No additional parameters are required to predict damage evolution. Also, as it is shown in this work, DDM parameters can be identified for Carbon fiber composites. This is not easily done for continuum damage mechanics (CDM) models because their state variables, namely the damage variables, are not measurable [19]. As a result, one is faced with identifying the model parameters using a macroscopic effect, such as the experimentally measured loss of stiffness, which for Carbon fiber composites is small [20]. Finally, DDM is available to be used in conjunction with commercial FEA environments such as Abaqus[1] [15] and ANSYS/Mechanical[2] [21], in the form of UMAT, UGENS [22], and USERMAT [23].

Therefore, the objective of this manuscript is to propose a methodology to determine values for the material properties required by the DDM model. In this work, the values for the parameters are found using available experimental data and a rational procedure. Once values are found, the DDM model is applied for predicting other, independent results, and conclusions are drawn about the applicability of the model.

No standard test method exist to measure the intralaminar fracture toughness. Although standards exist for measuring interlaminar fracture toughness in mode I (ASTM D5528) and proposed methods exists for mode II [24,25], intralaminar properties are

* Corresponding author.
E-mail address: drb@cemr.wvu.edu (E.J. Barbero).

[1] Abaqus and Simulia are trademarks or registered trademarks of Dassault Systèmes or its subsidiaries in the United States and/or other countries.
[2] Ansys® is a registered trademark of ANSYS Inc.

1359-8368/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compositesb.2013.08.084

not the same as interlaminar ones. Thus the need for a method to find the intralaminar fracture toughness using available data for a broad set of material systems.

2. Discrete damage mechanics

Given the crack density and the shell strain, DDM [18] updates the state variable, i.e., the crack density, and calculates the shell stress resultant and secant stiffness matrix A, B, D, and/or the tangent stiffness matrix, all of them functions of crack density. The crack density λ is an array containing the crack density for all the laminas at an integration point of the shell element. The strain refers to the shell strain array ϵ, κ, conjugate to the shell stress resultant array N, M. In this way, DDM is a constitutive model that can be implemented as a user material subroutine (UMAT, VUMAT, USERMAT) [23, usermatps-901] for flat plane stress elements and as a user general section (UGENS) for curved shell elements [22, ugens-std].

2.1. Damage initiation and evolution

Damage initiation and evolution are controlled by a single equation representing the Griffith's criterion for an intralaminar crack, i.e., the undamaging domain is defined by

$$g(\epsilon,\lambda) = max\left[\frac{G_I(\epsilon,\lambda)}{G_{IC}}, \frac{G_{II}(\epsilon,\lambda)}{G_{IIC}}\right] - 1 \leqslant 0 \tag{1}$$

where G_I, G_{II} are the strain energy release rates (ERR) in modes I and II, calculated with (15) and (16), and G_{IC}, G_{IIC} are the invariant material properties representing the energy necessary to create a new crack. We shall see that for fixed strain, both G_I, G_{II} are decreasing functions of λ. Therefore, (1) exhibits strain-hardening as a function of crack density λ, and thus stress-softening as a function of strain ϵ.

DDM calculates G_I, G_{II} using a micromechanics of damage model that reduces the 3D equilibrium equations

$$\frac{\partial \sigma_i}{\partial x_j} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}; \quad i,j = 1\ldots3 \tag{2}$$

to 2D using the following approximations. First, a state of plane stress for symmetric laminates under membrane loads allows us to eliminate the u_3 component of the displacement, by using the following

$$\sigma_3 = 0 \tag{3}$$

$$\frac{\partial u_3}{\partial x_i} = 0; \quad i = 1,2 \tag{4}$$

Then, (2) are recast in terms of the thickness average of the displacements in each lamina defined as follows

$$\hat{u}_i^{(k)} = \int_{-h_k/2}^{h_k/2} u_i(z)dz; \quad i = 1,2 \tag{5}$$

where h_k is the thickness of lamina k. Next, the intralaminar shear stress components τ_{j3}, with $j = 1, 2$, are assumed to vary linearly in each lamina

$$\tau_{j3}^{(k)}(x_3) = \tau_{j3}^{k-1,k} + \left(\tau_{j3}^{k,k+1} - \tau_{j3}^{k-1,k}\right)\frac{x_3 - x_3^{k-1,k}}{h_k}; \quad j = 1,2 \tag{6}$$

With these assumptions, the 3D equilibrium Eq. (2) reduce to a system of $2n$ partial differential equations in 2D, with 2 equations per lamina, in terms of displacements, where n is the number of laminas in the laminate.

Experimental [26] and theoretical considerations [1, Section 7.2.1] support the assumption of periodically spaced cracks that propagate suddenly, in a unstable fashion, through the thickness of the lamina and along the fiber direction. Therefore, the domain is that of a representative volume element (RVE) spanning the laminate thickness, between two adjacent cracks, as shown in Fig. 1. The length $2l$ of the RVE is inversely proportional to the crack density, i.e.,

$$\lambda = 1/2l \tag{7}$$

where $2l$ is the distance between two cracks.

In this way, the crack density enters the model through the length of the RVE. Since the ERR is computed in this RVE, which is independent of the finite element discretization, coupled to the fact that the constitutive model is formulated in terms of displacements rather than strains, the constitutive model is inherently mesh independent, without the need for choosing a characteristic length. Such mesh independence is corroborated by numerical results by plotting the reaction force vs. applied displacement on the boundary.

The PDE system is then solved with the following boundary conditions:

- Free stress boundary at the cracked surfaces. The surface of the cracks in lamina c, located at $x = \pm l$, are free boundaries, and thus subject to zero stress, with zero resultant force, as follows

$$h_c \int_{-1/2}^{1/2} \hat{\sigma}_j^{(c)}(x_1, l)dx_1 = 0; \quad j = 2,6 \tag{8}$$

where h_c is the thickness of the cracked lamina.

- Displacement compatibility. All laminas $m = 1\ldots n$ with n being the number of laminas, and $m \neq c$, that is, excluding the cracking lamina c, undergo the same displacement at the boundaries $(-l, l)$ when subjected to a membrane state of strain. Taking an arbitrary lamina $r \neq c$ as a reference, the remainder displacements are constrained as follows:

$$\hat{u}_j^{(m)}(x_1, \pm l) = \hat{u}_j^{(r)}(x_1, \pm l); \quad \forall m \neq c; \quad j = 1,2 \tag{9}$$

- Equilibrium. The stress resultant from the internal stress equilibrates the applied load.
  In the direction parallel to the surface of the cracks (fiber direction x_1) the load N_1 is supported by all the laminas

$$\frac{1}{2l}\sum_{k=1}^{N} h_k \int_{-l}^{l} \hat{\sigma}_1^{(k)}(1/2, x_2)dx_2 = N_1 \tag{10}$$

In the direction normal to the crack surface (x_2 direction) only the uncracked laminas $m \neq c$ carry normal and shear loads

$$\sum_{m \neq k} h_m \int_{1/2}^{1/2} \hat{\sigma}_j^{(m)}(x_1, l)dx_1 = N_j; \quad j = 2,6 \tag{11}$$

The solution of the PDE system yields the displacements in all laminas $\hat{u}_i^{(k)}$, and by differentiation, the strains in all laminas. Next,

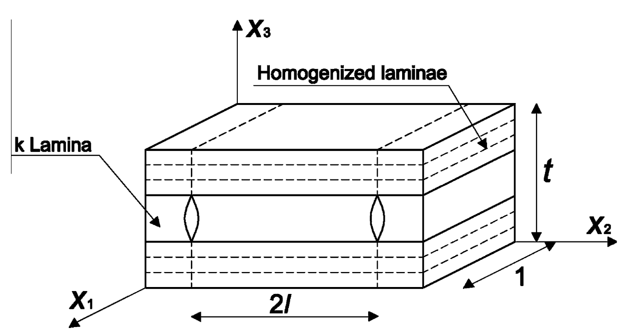

Fig. 1. Representative volume element between two adjacent cracks.