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Improved refined plate theory accounting for effect of thickness stretching in functionally graded plates

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ABSTRACT

In this paper, the refined pate theory is improved to account for the effect of thickness stretching in functionally graded plates. The refined plate theory has fewer number of unknowns and equations of motion than the first-order shear deformation theory, but accounts for the transverse shear deformation effects without requiring shear correction factors. The displacement field of the refined plate theory is modified by assuming a parabolic variation of the transverse displacement through the thickness, and consequently, the thickness stretching effect is taken into consideration. Closed-form solutions of simply supported rectangular plates are presented, and the obtained results are compared with 3D solutions and those predicted by higher-order shear deformation theories. Verification studies show that the proposed theory is not only more accurate than the refined plate theory, but also comparable with the higher-order shear deformation theories which contain more number of unknowns.

1. Introduction

The concept of functionally graded materials (FGMs) was first introduced in 1984 by material scientists in the Sendai area of Japan [1]. FGM is a class of composite materials that has continuous variation of material properties from one surface to another and thus eliminates the stress concentration found in laminated composites. Typically, the FGM is made from a mixture of a ceramic and a metal. FGMs are widely used in many structural applications such as mechanical, aerospace, civil, and automotive. When the application of FGMs increases, more accurate theories are required to predict their responses.

Since the shear deformation effects are more pronounced in thick plates or plates made of advanced composites like FGMs, shear deformation theories such as first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDTs) should be used to analyze functionally graded (FG) plates. The FSDT gives acceptable results, but requires a shear correction factor [2,3]. Whereas, the HSDTs [4–16] do not require a shear correction factor, but their equations of motion are more complicated than those of the FSDT. Therefore, Shimpi [17] has developed a refined plate theory (RPT) which is simple to use.

The RPT of Shimpi [17] accounts for a parabolic variation of the transverse shear strains through the thickness, and hence, a shear correction factor is not required. The displacement field of the RPT [17] is chosen based on the partition of the transverse displacements into the bending and shear parts. In fact, the idea of partitioning the transverse displacements into the bending and shear components is first proposed by Huffington [18], and later adopted by Krishna Murty [19] and Senthilnathan et al. [20]. The most interesting feature of the RPT [17] is that it contains fewer unknowns and governing equations than those of the FSDT and does not require a shear correction factor. Thus, it is the most efficient theory. The RPT was first developed for isotropic plates [17,21–23], and recently extended to orthotropic plates [24–28], laminated composite plates [29,30], laminated composite beams [31,32], FG plates [33–40], FG sandwich plates [41–43], nanobeams [44], and nanoplates [45–47]. It should be noted that the above-mentioned theories neglect the thickness stretching effect (i.e., $\varepsilon_z = 0$) by assuming a constant transverse displacement through the thickness of the plate. This assumption is appropriate for thin or moderately thick FG plates, but is inadequate for thick FG plates [48]. The effect of the thickness stretching in FG plates was studied by Carrera et al. [48], and it becomes significant in thick plates. Thus, it should be taken into consideration.

The purpose of this paper is to improve the RPT [17] by accounting for the effect of thickness stretching in FG plates. The displacement field of the RPT [17] is modified by assuming a parabolic variation of the transverse displacement through the thickness, and consequently, the thickness stretching effect is taken into consideration. The equations of motion are derived from Hamilton's principle. Analytical solutions for bending and vibration problems

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are obtained for a simply supported rectangular plate. Numerical examples are presented to verify the accuracy of the present study.

2. RPT accounting for thickness stretching effect

The RPT of Shimpi [17] is developed based on the following assumptions: (1) the in-plane and transverse displacements consist of bending and shear parts; (2) the bending parts of in-plane displacements are similar to those given by the classical plate theory; and (3) the shear parts of the in-plane displacements give rise to a parabolic variation of the shear strains, and hence to shear stresses through the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces. Based on the above assumptions, the displacement field of the RPT is obtained as [17]

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ u_2(x,y,z,t) &= v(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ u_3(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) \end{aligned} \tag{1}$$

where $f(z)=-z/4+5z^3/3h^2$; (u_1,u_2,u_3) are the the displacements along the (x,y,z) coordinate directions, respectively; u and v denote the displacements along the x and y coordinate directions of a point on the midplane of the plate; w_b and w_s are the bending and shear parts of the transverse displacement, respectively; h is the thickness of the plate. As mentioned previously, the RPT ignores the thickness stretching effect due to assuming a contant transverse displacement through the thickness. To account for the thickness stretching effect, the displacement field in Eq. (1) is modified by adding higher-order terms for the transverse displacement as

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ u_2(x,y,z,t) &= v(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ u_3(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) + g(z)w_z(x,y,t) \end{aligned} \tag{2}$$

where w_z is an unknown diplacement function accounting for the thickness stretching effect; and $g(z)$ is a shape function which is determined from the stress-free boundary conditions on the top and bottom surfaces of the plate. Using the same procedures presented by Reddy [49], the shape function $g(Z)$ is obtained as

$$g(z) = 1 - f'(z) = \frac{5}{4}\left(1 - \frac{4z^2}{h^2}\right) \tag{3}$$

The linear strains associated with the new displacement field in Eq. (2) are:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{4a}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2} - f(z)\frac{\partial^2 w_s}{\partial y^2} \tag{4b}$$

$$\varepsilon_z = g'(z)w_z \tag{4c}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w_b}{\partial x \partial y} - 2f(z)\frac{\partial^2 w_s}{\partial x \partial y} \tag{4d}$$

$$\gamma_{xz} = g(z)\left(\frac{\partial w_s}{\partial x} + \frac{\partial w_z}{\partial x}\right) \tag{4e}$$

$$\gamma_{yz} = g(z)\left(\frac{\partial w_s}{\partial y} + \frac{\partial w_z}{\partial y}\right) \tag{4f}$$

The stresses are obtained from the constitutive relations as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{5}$$

where C_{ij} are the 3D elastic constants given by

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)E}{(1-2\nu)(1+\nu)} \tag{6a}$$

$$C_{12} = C_{13} = C_{23} = \frac{\nu E}{(1-2\nu)(1+\nu)} \tag{6b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E}{2(1+\nu)} \tag{6c}$$

If the thickness stretching effect is omitted (i.e., $\varepsilon_z = 0$), the constitutive relations in Eq. (5) are rewritten as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{7}$$

where E and ν are Young's modulus and Poisson's ratio, respectively, of a FG plate. Since the effects of Poisson's ratio ν on the response of FG plates are very small [50,51], it is assumed to be constant for convenience. In this study, Young's modulus is assumed to vary through the plate thickness according to either the exponential or the power law distribution of the volume fractions of the constituents. According to the exponential distribution, Young's modulus $E(z)$ is given by [52]

$$E(z) = E_0 e^{p(0.5+z/h)} \tag{8}$$

where $E_m = E_0$ and $E_c = E_0 e^p$ denote Young's modulus of the bottom (as metal) and top (as ceramic) surfaces of the FG plate, respectively; E_0 is Young's modulus of the homogeneous plate; and p is a parameter that indicates the material variation through the plate thickness. According to the power law distribution with Mori–Tanaka scheme, the bulk modulus $K(z)$ is given by [53,54]

$$K(z) = K_m + (K_c - K_m)\frac{V_c}{1 + V_m \frac{K_c - K_m}{K_m + 4/3G_m}} \tag{9}$$

where subscripts m and c represent the metal and ceramic constituents, respectively; G is the shear modulus; and the volume fractions of the metal phase V_m and ceramic phase V_c are given by

$$V_m = 1 - V_c \text{ and } V_c = (0.5 + z/h)^p \tag{10}$$

Recall that the bulk and shear modulus are related to Young's modulus and Poisson ratio by $K = E/3(1-2\nu)$ and $G = E/2(1+\nu)$. Thus, by rewriting Eq. (9) in terms of E and ν, the effective Young's modulus $E(Z)$ is obtained as

$$E(z) = E_m + (E_c - E_m)\frac{V_c}{1 + V_m\left(\frac{E_c}{E_m} - 1\right)\frac{1+\nu}{3-3\nu}} \tag{11}$$

The mass density $\rho(z)$ is estimated using the power law distribution with Voigt's rule of mixtures as

$$\rho(z) = \rho_m V_m + \rho_c V_c \tag{12}$$

The strain energy of a plate can be expressed as