Composites: Part B 56 (2014) 733–739

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Analysis of effect of fiber orientation on Young's modulus for unidirectional fiber reinforced composites

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ARTICLE INFO

Article history:
Received 17 April 2013
Received in revised form 4 September 2013
Accepted 7 September 2013
Available online 17 September 2013

Keywords:
A. Glass fibers
B. Mechanical properties
C. Analytical modeling
C. Numerical analysis

ABSTRACT

Young's modulus of unidirectional glass fiber reinforced polymer (GFRP) composites for wind energy applications were studied using analytical, numerical and experimental methods. In order to explore the effect of fiber orientation angle on the Young's modulus of composites, from the basic theory of elastic mechanics, a procedure which can be applied to evaluate the elastic stiffness matrix of GFRP composite as an analytical function of fiber orientation angle (from 0° to 90°), was developed. At the same time, different finite element models with inclined glass fiber were developed via the ABAQUS Scripting Interface. Results indicate that Young's modulus of the composites strongly depends on the fiber orientation angles. A U-shaped dependency of the Young's modulus of composites on the inclined angle of fiber is found, which agree well with the experimental results. The shear modulus is found to have significant effect on the composites' Young's modulus, too. The effect of volume content of glass fiber on the Young's modulus of composites was investigated. Results indicate the relation between them is nearly linear. The results of the investigation are expected to provide some design guideline for the microstructural optimization of the glass fiber reinforced composites.

1. Introduction

Polymer materials reinforced with glass fiber have received tremendous attention in both scientific and industrial communities due to their extraordinary enhanced properties, such as lower weight, higher toughness and higher strength characteristics. In response to these requirements, research on composites has attracted much attention, which results in numerous publications [1–11].

In order to evaluate the mechanical behaviors of composites materials, different approaches, including experimental investigation, numerical simulations and theoretical modeling, were employed [12–16]. For example, the fiber bundle model has been used to study the damage behaviors of fiber while loading along the fibers [17]. The numerical continuum mechanical models, such as finite element models, allow the incorporation of many different features of the nonlinear material behaviors and the analysis of the interaction of available and evolving microstructural elements [18]. Many computational experiments of damage and failure in composites have been done by employing numerical continuum mechanical models [19–22]. The shear lag and other analytical models based on simplifying assumptions are applicable mainly to the linear elastic material behaviors and relatively simple, periodic microgeometrics. They are often used to analyze the load transfer and multiple cracking in composites [23]. The fracture mechanics-based models are often used to the cases of fiber bridging analysis of elastic or homogeneous material [24].

Additional, experimental investigation about mechanical behaviors of GFRP also made great advance. For example, SEM (scanning electron microscopy) in situ experiments of damage growth in GFRP composite under three-point bending loads were carried out. The dependence of mechanical parameters on the orientation angles of fibers was analyzed [25]. The tensile strength and fracture surface characterization of sized and unsized glass fibers were examined by single fiber tensile tests. The experimental tests clearly indicated that the unsized fibers were weaker in the low strength range, but had similar strength in the high strength range [26]. The interfacial shear strength between the fiber and the matrix of the fiber embedded matrix specimen was calculated by single fiber fragmentation test and fiber strengths of both sized and unsized fiber were found [27].

As a typical transverse isotropic material, the elastic properties of GFRP are characterized by five elastic constants. Fiber orientation with respect to loading direction is one of the most important parameters affecting mechanical properties of fiber reinforced composites. However, in case of Young's modulus, there is less reported on this aspect from theory, experiment or numerical

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1359-8368/$ - see front matter
http://dx.doi.org/10.1016/j.compositesb.2013.09.020

simulation. In the present work, theoretical analysis, finite element models as well as experimental investigations were used to study the stiffness, i.e. Young's modulus, of glass fiber reinforced composites. Composites cells with different glass fiber volume content were also simulated and the effect of shear modulus G on the Young's modulus was also analyzed.

2. Theoretical analysis

2.1. Transformation of strain and stress

Define the stress and strain vectors, in the coordinate system xoy, as shown in Fig. 1(a), as:

$$\sigma = [\sigma_{11}\sigma_{22}\sigma_{33}\tau_{12}\tau_{13}\tau_{23}]^T \tag{1}$$

$$\varepsilon = [\varepsilon_{11}\varepsilon_{22}\varepsilon_{33}\gamma_{12}\gamma_{13}\gamma_{23}]^T \tag{2}$$

The stiffness matrix in the coordinate system xoy is $\mathbf{D}$. Then the stress vector, strain vector and the stiffness matrix satisfy:

$$\sigma = \mathbf{D}\varepsilon \tag{3}$$

After a transformation, as shown in Fig. 1(c), we can obtain:

$$\sigma' = \mathbf{T_1}(\alpha)\sigma \tag{4}$$

$$\varepsilon' = \mathbf{T_2}(\alpha)\varepsilon \tag{5}$$

where,

$$\mathbf{T_1}(\alpha) = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & 0 & \sin 2\alpha & 0 & 0 \\ \sin^2\alpha & \cos^2\alpha & 0 & -\sin 2\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2}\sin 2\alpha & \frac{1}{2}\sin 2\alpha & 0 & \cos 2\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\alpha & \sin\alpha \\ 0 & 0 & 0 & 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \tag{6}$$

$$\mathbf{T_2}(\alpha) = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & 0 & -\frac{1}{2}\sin 2\alpha & 0 & 0 \\ \sin^2\alpha & \cos^2\alpha & 0 & \frac{1}{2}\sin 2\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \sin 2\alpha & -\sin 2\alpha & 0 & \cos 2\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\alpha & -\sin\alpha \\ 0 & 0 & 0 & 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{7}$$

Then in the coordinate system $x'o'y'$,

$$\sigma' = \mathbf{D}'(\alpha)\varepsilon' \tag{8}$$

where,

$$\mathbf{D}' = \mathbf{T_1}(\alpha)\mathbf{D}\mathbf{T_2}^{-1}(\alpha) \tag{9}$$

Thus, it is possible to evaluate the stiffness matrix of any fiber orientation angle, which is the angle between fiber direction and loading direction, according to transverse stiffness matrix for transverse isotropic material.

2.2. Calculation of transformed stiffness matrix

A special subclass of orthotropy is transverse isotropy, which is characterized by a plane of isotropy at every point in the material. GFRP composites could be considered as transverse isotropy material macroscopically. Assuming the 2–3 plane to be the plane of isotropy at every point, transverse isotropy requires $E_2 = E_3 = E_p$, $\nu_{12} = \nu_{13} = \nu_{tp}$, $\nu_{21} = \nu_{31} = \nu_{pt}$ and $G_{21} = G_{31} = G_t$, where p and t stand for "in-plane" and "transverse," respectively. Thus, while ν_{tp} has the physical interpretation of the Poisson's ratio that characterizes the strain in the plane of isotropy resulting from stress normal to it, ν_{pt} characterizes the transverse strain in the direction normal to the plane of isotropy resulting from stress in the plane of isotropy. In general, the quantities ν_{tp} is not equal to ν_{pt} and they are related by $\nu_{tp}/E_t = \nu_{pt}/E_p$. The stress–strain laws reduce to:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_t & -\nu_{pt}/E_p & -\nu_{pt}/E_t & 0 & 0 & 0 \\ -\nu_{tp}/E_t & 1/E_p & -\nu_p/E_p & 0 & 0 & 0 \\ -\nu_{tp}/E_t & -\nu_p/E_p & 1/E_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_t & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_p \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \tag{10}$$

where $G_p = E_p/2(1 + \nu_p)$ and the total number of independent constants is five.

The stability criterion requires that $E > 0$, $G > 0$ and $-1 < \nu < 0.5$. The value of E_t can be taken from experiment data. E_t took 15GPa in this study. Transverse isotropy materials have $E_t = E_{11} > E_{22} = E_{33} = E_p$, E_p was assumed to be 10 GPa. According to typical mechanical parameters for transverse isotropy material, we took $\nu_p = 0.25$, $\nu_{tp} = 0.3$, and $G_t = 2$ GPa. Substituting these values into the flexibility matrix, then

$$\mathbf{C} = \begin{bmatrix} 0.067 & -0.02 & -0.02 & 0 & 0 & 0 \\ -0.02 & 0.1 & -0.025 & 0 & 0 & 0 \\ -0.02 & -0.025 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.25 \end{bmatrix} \tag{11}$$

The stiffness matrix $\mathbf{D}$ will be:

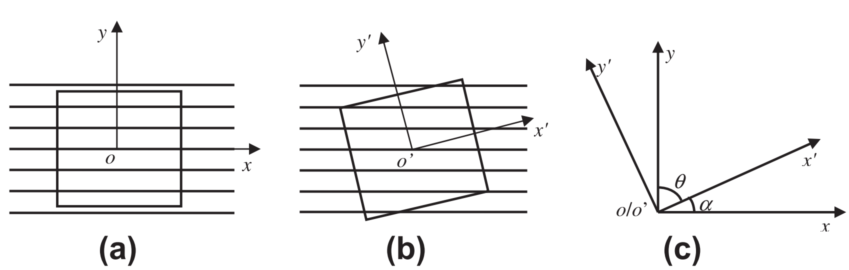

Fig. 1. Definition of two coordinate systems.