



Modeling the non-linear deformation of a short-flax-fiber-reinforced polymer composite by orientation averaging



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ABSTRACT

The growing usage of short-flax-fiber-reinforced polymer composites in such applications as automotive industry necessitates the prediction of their mechanical response up to and beyond the limit of elasticity. Due to the imperfect, mechanical interlocking-dominated adhesion of natural fibers to most polymers, both fiber debonding and matrix yielding contribute to the non-linear deformation. In the present study, the deformation under an active loading of a short misaligned fiber composite is modeled by the orientation averaging approach, employing an analytical description of the behavior of a unit cell (UC), the parameters of which are determined using a FEM analysis of UC response under selected loading modes. The model is applied to the prediction of stress–strain diagrams in tension of flax/polypropylene composites with different fiber volume fractions.

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1. Introduction

Short-natural-fiber-reinforced polymer composites have found applications in diverse non-structural components, e.g. in automotive industry [1,2]. Their deformation under service loads is governed by the mechanical properties of the constituents, as well as fiber length and orientation distributions. Several methods of different accuracy and complexity have been elaborated for evaluation of the elastic properties of unidirectionally aligned (UD) short-fiber composites [3]. The effect of fiber misalignment can then be allowed for by orientation averaging. The simplest method of averaging is based on the Voigt, or equistrain assumption (see, e.g. [3]). The average stress σ_{ij}^c in a misaligned-fiber composite under an applied strain ϵ_{ij}^c is then given by:

$$\sigma_{ij}^c(\epsilon^c) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi \sigma_{ij}^{UD}(\epsilon^c, \varphi, \theta) f(\varphi, \theta) \sin \theta d\varphi d\theta \quad (1)$$

where $f(\varphi, \theta)$ designates the fiber orientation distribution density as a function of the azimuthal, φ , and elevation, θ , angles and $\sigma_{ij}^{UD}(\epsilon^c, \varphi, \theta)$ stands for the stress under a given applied strain in a UD-reinforced computational element of the composite. The latter can be interpreted as a unit cell (UC) or a UD short-fiber composite with the same fiber volume fraction as that of the misaligned-fiber composite.

For linear elastic composites, Eq. (1) reduces to averaging the stiffness tensor of a UD short-fiber composite according to the actual fiber orientation distribution in the misaligned composite. This procedure has been shown to produce reasonably accurate estimates for both inorganic and natural fiber composites, see e.g. [4,5]. Furthermore, the accuracy of the approach has been verified by numerical simulations [6,7]. Moreover, using the average fiber length in modeling provides a sufficient accuracy since the actual fiber length distribution has been demonstrated to exert only a limited effect on the predicted composite properties [8].

Direct application of the orientation averaging technique in the case of non-linear response of the composite is complicated by the necessity of determining the UC response under complex loadings, which is usually done numerically [9,10]. As an alternative, the analytical relations developed in [11] for active loading of an inelastic anisotropic material can be applied. Then only a limited number of model parameters have to be determined for the UC to predict its response under arbitrary complex loading, thus simplifying considerably the averaging procedure stipulated by Eq. (1).

In the current study, we apply the orientation averaging method to predicting of the non-linear deformation of short-flax-fiber-reinforced polypropylene composites in tension. The analytical model proposed in [11] is used to describe the deformation of a UC, consisting of a fiber embedded in the polymer matrix, in active combined loading, to be averaged according to Eq. (1). The model parameters are determined using deformation diagrams of the UC under simple loading modes obtained by means of FEM

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simulations. The effect of imperfect adhesion of the fibers on the non-linear deformation of the composite is considered.

2. Unit cell model

In the following, we briefly present an analytical model for description of the non-linear deformation of an anisotropic composite material in complex active loading, elaborated in [11]. Further, in Section 2.2, we provide the procedure of evaluation of the model parameters of a UC of the short-flax-fiber composite based on deformation diagrams of the UC in simple loading modes. The latter are obtained by using the FEM model of UC described in Section 2.3.

2.1. Analytical description of deformation of the unit cell

For description of the nonlinear behavior of a composite UC, the geometry of which is given in Section 2.3, we use the form proposed in [11]:

$$\varepsilon_{ij} = a_{ijkl} \sigma_{kl} \frac{1}{kp} \tan(kp) \quad (2)$$

where a_{ijkl} is the compliance tensor and k is a numerical constant. The scalar function p is defined as:

$$p = p(\boldsymbol{\sigma}) = h(\boldsymbol{\sigma}) (b_{ijkl} \sigma_{ij} \sigma_{kl})^{\frac{1}{2}} \quad (3)$$

Here, b_{ijkl} designates a fourth-rank tensor with the following symmetries $b_{ijkl} = b_{jikl} = b_{ijlk} = b_{klij}$, and

$$h = h(\boldsymbol{\sigma}) = \frac{1 + c_1 \omega}{1 + c_2 |\omega|} \quad (4)$$

with the auxiliary parameter ω given by:

$$\omega = \frac{(a_{ik} a_{jm} a_{ln} \sigma_{ij} \sigma_{kl} \sigma_{mn})^{\frac{1}{3}}}{(a_{ik} a_{jl} \sigma_{ij} \sigma_{kl})^{\frac{1}{2}}} \quad (5)$$

where a_{ij} are defined as $a_{ij} = a_{ijmm}$, and the parameters c_1 and c_2 are such that $c_1 \omega > -1$, $c_2 \geq 0$.

Solving Eq. (2) for stresses, one obtains [11]:

$$\sigma_{ij} = A_{ijkl} \varepsilon_{kl} \frac{1}{kp} \arctan(kp) \quad (6)$$

where A_{ijkl} is the stiffness tensor,

$$P = P(\boldsymbol{\varepsilon}) = H(\boldsymbol{\varepsilon}) (B_{ijkl} \varepsilon_{ij} \varepsilon_{kl})^{\frac{1}{2}}$$

with $B_{ijkl} = A_{ijmn} b_{mnop} A_{opkl}$ and $H(\boldsymbol{\varepsilon}) = h(A_{ijkl} \varepsilon_{kl}) = h(\boldsymbol{\sigma})$.

Thus the non-linear deformability of the UC is characterized, in the general case, by five independent components of the tensor b_{ijkl} and three scalar parameters k , c_1 and c_2 .

2.2. Parameter estimation

We focus here only on the UC model parameters characterizing the non-linear deformation, since the elastic properties of the UC can be evaluated separately, by analytical or numerical models [3,5]. Therefore its compliance tensor a_{ijkl} is considered as known. The UC is assumed to be transversely isotropic, with the longitudinal principal axis along the fiber direction designated by 1.

In the case of linear elastic response of the UC in the longitudinal direction (i.e. fiber direction), the method of parameter estimation proposed in [11] can be used. It employs three stress–strain diagrams of the UD composite: in transverse tension and compression,

$\varepsilon_{22}(\sigma_{22})$, and pure shear, $\varepsilon_{12}(\sigma_{12})$, and the model parameters are obtained as the values providing the best fit of analytical relation Eq. (2) to the diagrams.

If the behavior of a composite material is nonlinear also in the longitudinal direction, more input data are needed. In addition to the three loading cases mentioned above, we also selected the axial tension $\varepsilon_{11}(\sigma_{11})$, shear $\varepsilon_{23}(\sigma_{23})$ and equi-biaxial tension $\varepsilon_{22}(\sigma_{11}, \sigma_{22})$ with $\sigma_{11} = \sigma_{22} = \sigma$. The deformation diagrams mentioned are to be obtained by the FEM, using the UC model described in Section 2.3.

Since the tensor b_{ijkl} is defined with accuracy up to a multiplicative constant, we are free to arbitrarily select the value of one of its components. Here we assign $b_{1212} = 1 \text{ GPa}^{-1}$, as suggested in [11]. Then the value of k can be determined by approximation of the shear response curve $\varepsilon_{12}(\sigma_{12})$ by Eq. (2). Under pure shear, Eq. (5) yields $\omega = 0$, and hence $h = 1$ according to Eq. (4), therefore Eq. (2) reads:

$$\varepsilon_{12} = a_{1212} \sigma_{12} \frac{1}{k \sqrt{b_{1212} |\sigma_{12}|}} \tan(2k \sqrt{b_{1212} |\sigma_{12}|}) \quad (7)$$

Similarly, b_{2323} is determined by approximation of the curve $\varepsilon_{23}(\sigma_{23})$ by Eq. (2), which reads in this case:

$$\varepsilon_{23} = a_{2323} \sigma_{23} \frac{1}{k \sqrt{b_{2323} |\sigma_{23}|}} \tan(2k \sqrt{b_{2323} |\sigma_{23}|}) \quad (8)$$

The value of ω is close to unity for equibiaxial tension; allowing for that, the remaining parameters and tensor b_{ijkl} components are determined assuming $\omega = 1$ for equi-biaxial tension. It is straightforward to check that h takes the same, constant, value for the axial longitudinal $\varepsilon_{11}(\sigma_{11})$, transverse $\varepsilon_{22}(\sigma_{22})$, and equi-biaxial $\varepsilon_{22}(\sigma_{11}, \sigma_{22})$ tension. Then the auxiliary parameters $C_1 = h \sqrt{b_{1111}}$ and $C_2 = h \sqrt{b_{2222}}$ are determined by approximation of the curves $\varepsilon_{11}(\sigma_{11})$ and $\varepsilon_{22}(\sigma_{22})$ by Eq. (2). This provides the ratio of tensor components

$$\frac{b_{2222}}{b_{1111}} = \left(\frac{C_2}{C_1} \right)^2 \quad (9)$$

The parameter $C_3 = h \sqrt{b_{1111} + 2b_{1122} + b_{2222}}$ is determined by approximation of the curve $\varepsilon_{22}(\sigma_{11}, \sigma_{22})$; the ratio of C_3 and C_1 yields

$$\frac{b_{1111} + 2b_{1122} + b_{2222}}{b_{1111}} = \left(\frac{C_3}{C_1} \right)^2 \quad (10)$$

Eqs. (9) and (10), supplemented by the assumptions of linear volume variation with stress under hydrostatic loading [11]:

$$b_{1111} + 2b_{2222} + 4b_{1122} + 2b_{2233} = 0 \quad (11)$$

and transverse isotropy of the UC,

$$b_{2222} - b_{2233} = 2b_{2323} \quad (12)$$

allow estimation of all the remaining components of the tensor b_{ijkl} from the system of linear equations Eqs. (9)–(12). Then the value of h in uniaxial tension, designated in the following as \bar{h} , is also obtained employing e.g. the auxiliary parameter C_1 .

To evaluate the parameters c_1 and c_2 entering Eq. (4), the transverse compression diagram $\varepsilon_{22}(\sigma_{22})$ is used. In this case, Eq. (2) becomes:

$$\varepsilon_{22} = a_{2222} \frac{\text{sgn}(\sigma_{22})}{k \bar{h} \sqrt{b_{2222}}} \tan(k \bar{h} \sqrt{b_{2222} |\sigma_{22}|}) \quad (13)$$

where \bar{h} designates the value of h in uniaxial compression. Approximating the $\varepsilon_{22}(\sigma_{22})$ diagram by Eq. (13), \bar{h} is determined.

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