Nuclear Instruments and Methods in Physics Research A 694 (2012) 101–106

Contents lists available at SciVerse ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

Implementation of a truncated cusp filter for real-time digital pulse processing in nuclear spectrometry

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ARTICLE INFO

Article history:
Received 29 June 2011
Received in revised form
20 July 2012
Accepted 23 July 2012
Available online 9 August 2012

Keywords:
Digital filters
Truncated cusp
Trapezoid
FPGA
Resolution
Detectors

ABSTRACT

In this work the transfer function for a truncated cusp infinite impulse response (IIR) filter has been derived. The truncated cusp filter, when given an exponential pulse in the presence of white noise, outputs a truncated cusp pulse which is optimum pulse shape under the constraint of finite pulse duration. However it features a pole outside of the unit-circle and is therefore inherently unstable. To overcome this instability a switching algorithm with two alternating poles has been applied. The filter has been simulated with MATLAB and Simulink and implemented on-board a FPGA. The pulses from a waveform generator and silicon drift detector (SDD) have been also processed by the filter and results compared with theory. We find that the simulation and measurements were in agreement and show that energy resolution due to electronic noise can be improved by up to 7.5% using a truncated cusp instead of triangular filter. An improvement of up to 2.8% in overall energy resolution was measured with a typical X-ray spectrometer based on a SDD detector with a nominal energy resolution of 150 eV at energies of MnK_α line.

1. Introduction

The energy resolution of semiconductor radiation detectors can be affected by its subsequent electronic processing chain. This chain is used to measure charge corresponding to a current pulse of known shape. Charge–hole pairs are created by incident radiation and current is induced by the bias voltage across the detector. The signal processing chain usually comprises two stages. The first stage is a trans-impedance amplifier that converts the current pulse into a voltage signal. The second is a pulse shaping amplifier that filters out noise in order to get a resolution as close as possible to the intrinsic resolution of the detector [1–3].

The time-domain signal shape at the output of the trans-impedance amplifier with capacitance in the feedback path is as equally close to a step function as the current pulse is close to the delta function. At the same place in the chain, noise can be modeled by three different frequency components: white, color and flicker with a power spectral density (PSD) according to [1–3]

$$N(\omega) = a + \frac{b}{\omega^2} + \frac{c}{\omega}, \tag{1}$$

where constants a, b and c depend on the detector and the trans-impedance amplifier characteristics.

The problem of which filter provides the best signal-to-noise ratio can be solved by splitting the pulse shaping stage into two filters [4]. The first is a whitening filter that makes the noise PSD independent of frequency. Then a second, shaping filter, which has impulse response, according to matched filter theory, equal to the mirror image of the output signal from the whitening filter with respect to time, is applied.

Under the condition that flicker noise can be neglected, the whitening filter is a simple high-pass filter (HPF) with corner time constant

$$\tau_c = \sqrt{\frac{a}{b}} \tag{2}$$

Since the shape of the input signal into the whitening filter is a step function, the output is an exponential signal with a decay time equal to the corner time constant. Therefore, the impulse response of the shaping filter is a mirrored exponential function with respect to time. The signal on its output can be synthesized by taking a convolution of its input signal with its impulse response. This gives the well-known signal with infinite cusp shape. When normalized to unit amplitude, the signal has the following form:

$$s_\infty(t) = e^{-\frac{|t|}{\tau_c}} \tag{3}$$

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http://dx.doi.org/10.1016/j.nima.2012.07.042

2. Synthesis of the truncated infinite impulse response (IIR) cusp filter

The second shaping stage, called cusp filter, which inputs an exponential signal with decay-time constant τ_c, and outputs an infinite cusp is not realizable. It can be shown [4] that the optimal realizable limited time filter is one with output signal of a truncated cusp shape. Normalized to unit amplitude, the optimal signal QUOTE has the following form [3,5]:

$$s(t)=\begin{cases}\frac{\sinh\frac{t}{\tau_c}}{\sinh\frac{\tau_{pk}}{\tau_c}} & 0<t<\tau_{pk}\\ \frac{\sinh\frac{2\tau_{pk}-t}{\tau_c}}{\sinh\frac{\tau_{pk}}{\tau_c}} & \tau_{pk}<t<2\tau_{pk}\end{cases}, \tag{4}$$

where τ_{pk}, the peaking time is half the width of the signal. Its signal-to-noise ratio η_{tcs} compared to infinite cusp's, η_∞, is given by the following relation (see Appendix, Eq. A8):

$$\frac{\eta_{tcs}}{\eta_\infty}=\tanh\left(\frac{\tau_{pk}}{\tau_c}\right) \tag{5}$$

In order to synthesize the truncated cusp filter in the digital domain, the input exponential signal should be digitized. If the digitization period is T then the input signal $s_i(n)$ with unit amplitude can be described in the digital domain by the following:

$$s_i(n)=e^{-\frac{nT}{\tau_c}}h(n), \tag{6}$$

where $h(n)$ is the Heaviside step function. The truncated cusp filter should output a signal with a shape equal to Eq. (4) and should be shifted by τ_{pk} to synchronize it with the input (6). In order to minimize any possible ballistic deficit, a flat top [3,5] can be added to the output signal but at the expense of decreasing the signal-to-noise ratio. This gives the following shape of the filter's output signal $S_o(n)$ in the digital domain:

$$\begin{aligned}s_o(n)=&\frac{\sinh\frac{nT}{\tau_c}}{\sinh\frac{mT}{\tau_c}}(h(n)-h(n-m))\\&-\frac{\sinh\frac{(n-l-m)T}{\tau_c}}{\sinh\frac{mT}{\tau_c}}(h(n-l)-h(n-l-m))\\&+h(n-m)-h(n-l),\end{aligned} \tag{7}$$

where m is equal to peaking time (τ_{pk}) and $l-m$ to the flat top duration, both in clock period units.

By knowing the input and output signal shape one can calculate the transfer function $H(z)$ by using the well known $\mathcal{Z}$ transform. The calculation gives the following form:

$$\begin{aligned}H(z)=&\frac{\sinh\frac{T}{\tau_c}}{\sinh\frac{mT}{\tau_c}}\frac{\left(1-e^{\frac{mT}{\tau_c}}z^{-m}\right)\left(1-e^{-\frac{mT}{\tau_c}}z^{-l}\right)}{z-e^{\frac{T}{\tau_c}}}\\&+\left(1-e^{-\frac{T}{\tau_c}}\right)\frac{z^{-m}-z^{-l}}{z-1}\end{aligned} \tag{8}$$

Note that the second term in Eq. (8) vanishes for zero-time flat-top, i.e. $m=l$.

3. Implementation of the truncated IIR cusp filter

A straightforward implementation of the filter given by the transfer function (8) fails numerically in part due to a term $H_{\mathrm{TIIR}}(z)$which has a pole $p=e^{T/\tau_c}$, outside of the unit-circle. The term $H_{\mathrm{TIIR}}(z)$is given by the expression

$$H_{\mathrm{TIIR}}(z)=\frac{1-p^m z^{-m}}{1-pz^{-1}} \tag{9}$$

This so called one-pole Truncated Infinite Impulse Response (TIIR) filter, has been studied elsewhere [6] and a simple solution proposed for its implementation: two alternating instances of the TIIR filter. In this work we have followed the same approach. The filter layout in the MATLAB/Simulink [7] and Xilinx System Generator [8] graphical environment is shown in Fig. 1. It can be seen that H_{TIIR} comprises one shared delay line and two one-pole filters. Before input into H_{TIIR}, the signal was scaled by an additional factor, $\exp(T/\tau_c)$, apart from the normalization factor $\sinh(T/\tau_c)/\sinh(mT/\tau_c)$in Eq. (8), such that the amplitude of the output signal from $H_{\mathrm{TIIR}}$is normalized to unitarity. If H_{TIIR} is calculated in M-bit fixed point arithmetic precision and term p^m represented with N fractional bits, then a maximal value for peaking times is approximately equal to $\tau_c\ln\left(2^{M-N}\right)$. It has been verified that calculation of H_{TIIR} with the proposed algorithm and using $M=32$ and $N=26$ is sufficiently precise for broad range of peaking times.

Details of the original switching algorithm and analysis of the error propagations can be found in Ref. [9]. In this work the original switching algorithm was modified in such a way that the switching can be done every $m+1$ clock cycle. It makes the filter run shorter hence it accumulates less errors. The switching is done on the following way. When output of the first pole '1p1' on Fig. 1 is selected, then its accumulator starts receiving output from the subtractor 'sub1'. In the same time accumulator of the second pole '1p2' is reset and it starts receiving earlier input 'a' of the same subtractor. This phase lasts for $m+1$ clock cycles including one clock cycle needed to reset accumulator. After that, poles are switched: '1p1' is reset and starts receiving earlier input 'a' of the subtractor 'sub1' while '1p2' starts receiving output from the same subtractor. In this phase output is taken from pole '1p2'. The filter was running at 50 MHz clock and its implementation required 24 18×18 bit embedded multipliers.

A FIR implementation of Eq. (9) contains m sum of products and requires m multipliers. For example, a filter with peaking time $\tau_{pk}=10\ \mu s$ which runs at $f=50$ MHz would require $m=f\tau_{pk}=500$ multipliers. Therefore the IIR form (8) of a truncated cusp transfer function and its corresponding recursive relation is very suitable for a low-cost and low-power FPGA, which have a lack of embedded multipliers.

The output from H_{TIIR} feeds the part of the filter described by the last factor in the first term of Eq. (8). In the case when a flat top is required the second term in Eq. (8) should be implemented and added to the first term.

4. Simulation and measurement

In order to verify the switching algorithm and check the efficiency of the filter, a simulation in Simulink was performed. For rapid simulation, the filter was synthesized using Simulink blocks only and the simulation clock period was reduced to 10 MHz. The outputs from the trans-impedance amplifier were simulated by periodically superimposing noiseless step signals of the same amplitude. This created a noiseless staircase signal which fed the HPF with corner time constant $\tau_c=5\ \mu s$. Thereafter white noise was added to the output of the HPF and the summing signal was sampled and held by Simulink's 'Hold' block and converted with the 'Idealized ADC quantizer'. For each peaking time, the output signal from the filter was stored into the Work-space and later analyzed. In the analysis the height of each pulse was calculated simply by taking its maximal amplitude. In this way, the pulse height distribution versus peaking time was obtained. The simulated distributions were in very good agreement with the expected Gaussian. The full width at half maximum (FWHM) of the above Gaussians was plotted with respect to peaking time τ_{pk}. By definition, the calculated FWHM should be inversely proportional to the square root of the signal-to-noise ratio η given by Eq. (5). When normalized to the estimated FWHM