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Multi-line enhanced beam model for the analysis of laminated composite structures

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ABSTRACT

A novel approach to the analysis of composite structure is introduced in this paper. One-dimensional (1D) refined finite elements are formulated by making use of the Carrera Unified Formulation (CUF). CUF is a higher-order 1D formulation which was recently introduced by the first author. By exploiting the hierarchical characteristics of CUF, a multi-line approach is developed straightforwardly and used for the analysis of multilayered structures. In the multi-line approach, each layer is modeled by one beam-line discretization. Refined beam elements with different orders of expansion over the cross-sectional plane are then employed along different beam-lines. The compatibility of displacements at the boundary interfaces between beam-lines is ensured by using Lagrange multipliers. The accuracy of the proposed method is verified both through published literature and through finite element solutions using the commercial code MSC/Nastran.

1. Introduction

This paper is devoted to the analysis of laminated composite beams. The advantages of composite materials are well known and the most relevant are: high strength-to-weight ratio, high stiffness-to-weight ratio, ease of formability, wide range of operating temperatures, and their capability to be tailored according to a given requirement (see the book by Tsai [1]). One of the main issues related to the proper modeling of a composite structure is related to its low transverse shear moduli compared to the axial tensile moduli, as discussed in the excellent review of Kapania and Raciti [2,3] which includes a comprehensive overview on composite beam works. Moreover, the characterization of anisotropic layered composite structures requires models able to reproduce piecewise continuous displacement and transverse stress fields in the thickness direction. These two effects, summarized as C_z^0 requirements in [4], are not automatically satisfied by those models that were originally devoted to the analysis of single-layered structures, such as the classical beam theories by Euler [5] (hereinafter referred to as EBBM) and Timoshenko [6] (hereinafter referred to as TBM).

A great deal of literature exists on classical and refined beam theories for the analysis of multilayered composite structures. A brief, though not exhaustive review, is given hereafter. Reddy [7] presented a plate theory which provides a parabolic distribution of the transverse shear strains ensuring that the transverse shear stresses are null on the top and bottom surfaces. By using this model, exact closed-form solutions for static analyses of cross-ply laminated beams with arbitrary boundary conditions were presented in [8]. In [9], Surana and Nguyen presented an interesting two-dimensional hierarchical curved beam element. In Matsunaga's paper [10], the displacement components were expanded into power series of the *z*-thickness coordinate. Mantari et al. [11] expressed the displacement components of laminated plates by adopting a combination of exponential and trigonometric functions. Recently, Vidal et al. [12] proposed the approximation of the displacement field as a sum of separated functions of axial and transverse coordinates by adopting the Proper Generalized Decomposition procedure.

All the aforementioned theories are based on the Equivalent Single layer (ESL) approach and, although the results agree very well with the three-dimensional solutions for several structural problems, the main drawback is the continuity of shear strains at interfaces (hence the discontinuity of the shear stresses if they are computed through the constitutive equations). To overcome this shortcoming, many researchers have adopted the Layer-wise (LW) approach and a few examples are given here. Shimpi and Ghugal [13] presents a new Layer-wise (LW) trigonometric model for two-layered cross-ply beams. The main feature of this theory is that the shear stresses are derived directly from the constitutive equations satisfying both the shear-stress-free condition at the free surfaces of the beam and the condition of continuity of the shear

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stresses at the interface. On the same topic, Tahani [14] proposes two theories to analyze the static and dynamic behavior of the laminated beams. Unfortunately, when the number of layers increases, the LW approach becomes unfavorable because it is too expensive in terms of computational cost. To overcome this problem, many researchers have introduced layer independent theories in which zig-zag or Heaviside's functions are used.

Murakami [15] was the first to introduce a zig-zag function into Reissner's new mixed variational principle to develop a plate theory (for a comprehensive review of Murakami's zig-zag method, see Carrera [16]). Vidal and Polit [17] presented a refined sine model by exploiting a Heaviside function for each layer to satisfy the continuity conditions for both displacements and transverse shear stress and the free conditions of the upper and lower surfaces. An extensive investigation about the use of various cross-sectional functions for the analysis of laminated beams has recently been proposed in [18], where polynomial, trigonometric, exponential, as well as any combination of these functions were used.

It is clear that many attempts have been made in order to provide a general and reliable theory able to capture every aspect of the complex nature of composite materials. In the present paper, a new method for the analysis of laminated composite structures is proposed. This method, which is called *Multi-Line* (ML), represents a step forward from the classical ESL and LW approaches. ML models have recently been introduced by Carrera and Pagani [19] and used for the analysis of thin-walled and reinforced structures. In the present paper ML approach is extended to the analysis of composite structures. In a ML modeling approach for laminates, each layer (or group of layers) of the structure is modeled by one higher-order beam. Subsequently, higher-order beams are assembled at the layer interfaces through Lagrange multipliers. In this work, refined beam elements are formulated using the Carrera Unified Formulation (CUF). According to CUF, Taylor-like polynomials are used on the cross-section of each beam to expand generalized displacement variables in the neighborhood of the beam axis. CUF was originally devoted to the analysis of plate and shell structures [20] and recently it has been expanded to 1D theories by the first author and his co-workers [21]. Several papers are available on the analysis of composite structures via CUF models, see for example [18,22–24].

In the next sections a brief overview on CUF and the ML approach is provided. Numerical results concerning laminated and composite structures are then discussed. Finally, the main conclusions are outlined.

2. Higher-order beam formulation

2.1. Preliminaries

The adopted rectangular cartesian coordinate system is shown in Fig. 1, together with the geometry of a beam which can be considered as a single layer, a group of layers, as well as a whole multilayer. The cross-sectional plane of the structure is denoted by Ω, and the beam boundaries over y are $0 \leqslant y \leqslant l$. Subscript k, which is usually used to denote variables and parameters related to the kth

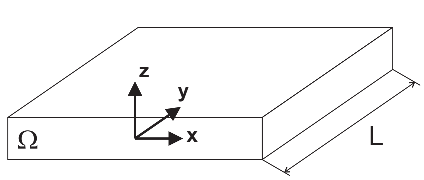

Fig. 1. Coordinate frame of the beam model.

layer, is neglected in the following for the sake of simplicity. Let us introduce the transposed displacement vector,

$$\mathbf{u}(x,y,z) = \{ u_x \quad u_y \quad u_z \}^T \tag{1}$$

The stress, $\boldsymbol{\sigma}$, and strain, $\boldsymbol{\epsilon}$, components are grouped as follows:

$$\begin{aligned} \boldsymbol{\sigma}_p &= \{ \sigma_{zz} \quad \sigma_{xx} \quad \sigma_{zx} \}^T, \quad \boldsymbol{\epsilon}_p = \{ \epsilon_{zz} \quad \epsilon_{xx} \quad \epsilon_{zx} \}^T \\ \boldsymbol{\sigma}_n &= \{ \sigma_{zy} \quad \sigma_{xy} \quad \sigma_{yy} \}^T, \quad \boldsymbol{\epsilon}_n = \{ \epsilon_{zy} \quad \epsilon_{xy} \quad \epsilon_{yy} \}^T \end{aligned} \tag{2}$$

In the case of small displacements with respect to a characteristic dimension in the plane of Ω, the linear strain–displacement relations are used.

$$\begin{aligned} \boldsymbol{\epsilon}_p &= \mathbf{D}_p \mathbf{u} \\ \boldsymbol{\epsilon}_n &= \mathbf{D}_n \mathbf{u} = (\mathbf{D}_{n\Omega} + \mathbf{D}_{ny})\mathbf{u} \end{aligned} \tag{3}$$

where $\mathbf{D}_p$ and $\mathbf{D}_n$ are linear differential operators and the subscript "n" stands for terms lying on the cross-section, while "p" stands for terms lying on planes which are orthogonal to Ω.

$$\mathbf{D}_p = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix}, \quad \mathbf{D}_{n\Omega} = \begin{bmatrix} 0 & \frac{\partial}{\partial z} & 0 \\ 0 & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D}_{ny} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \end{bmatrix} \tag{4}$$

Constitutive laws are now exploited to obtain stress components to give

$$\boldsymbol{\sigma} = \tilde{\mathbf{C}}\boldsymbol{\epsilon} \tag{5}$$

Eq. (5) can be split into $\boldsymbol{\sigma}_p$ and $\boldsymbol{\sigma}_n$ with the help of Eq. (2) so that

$$\begin{aligned} \boldsymbol{\sigma}_p &= \tilde{\mathbf{C}}_{pp}\boldsymbol{\epsilon}_p + \tilde{\mathbf{C}}_{pn}\boldsymbol{\epsilon}_n \\ \boldsymbol{\sigma}_n &= \tilde{\mathbf{C}}_{np}\boldsymbol{\epsilon}_p + \tilde{\mathbf{C}}_{nn}\boldsymbol{\epsilon}_n \end{aligned} \tag{6}$$

The matrices $\tilde{\mathbf{C}}_{pp}$, $\tilde{\mathbf{C}}_{nn}$, $\tilde{\mathbf{C}}_{pn}$, and $\tilde{\mathbf{C}}_{np}$ contains the material coefficients. They can be found in [25] in the case of orthotropic material, which is considered in this work.

Within the framework of CUF, the displacement field $\mathbf{u}(x,y,z)$ can be expressed as

$$\mathbf{u}(x,y,z) = F_\tau(x,z)\mathbf{u}_\tau(y), \quad \tau = 1,2,\ldots,M \tag{7}$$

where F_τ are the functions of the coordinates x and z on the cross-section. $\mathbf{u}_\tau$ is the vector of the *generalized* displacements, M stands for the number of terms used in the expansion, and the repeated subscript, τ, indicates summation. The choice of F_τ determines the class of the 1D CUF model that is required and subsequently to be adopted. TE (Taylor expansion) 1D CUF models – described by Eq. (7) – consists of a Maclaurin series that uses the 2D polynomials $x^i z^j$ as base, where i and j are positive integers. For instance, the displacement field of the second-order ($N = 2$) TE model can be expressed as

$$\begin{aligned} u_x &= u_{x_1} + x\, u_{x_2} + z\, u_{x_3} + x^2\, u_{x_4} + xz\, u_{x_5} + z^2\, u_{x_6} \\ u_y &= u_{y_1} + x\, u_{y_2} + z\, u_{y_3} + x^2\, u_{y_4} + xz\, u_{y_5} + z^2\, u_{y_6} \\ u_z &= u_{z_1} + x\, u_{z_2} + z\, u_{z_3} + x^2\, u_{z_4} + xz\, u_{z_5} + z^2\, u_{z_6} \end{aligned} \tag{8}$$

The order N of the expansion is set as an input option of the analysis; the integer N is arbitrary and defines the order of the beam theory. The Timoshenko beam model (TBM) can be realized by using a suitable F_τ expansion. Two conditions have to be imposed: (1) a first-order ($N = 1$) approximation kinematic field and (2) the displacement components u_x and u_z have to be constant above the cross-section. By contrast, the Euler–Bernoulli beam model (EBBM) can be obtained through the penalization of ϵ_{xy} and ϵ_{zy}. Classical theories and first-order models ($N = 1$) require the necessary assumption of reduced material stiffness coefficients to correct Poisson's locking (see [26]). In this paper, Poisson's locking is corrected according to