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Technical Notes

# Kinematics analysis of six-bar parallel mechanism and its applications in synchrotron radiation beamline

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ABSTRACT

Six-bar parallel mechanism is now widely applied in synchrotron radiation beamline, while the six-dimensional adjustment is difficult and inefficient for lack of theoretical direction. This paper introduces a special six-bar parallel mechanism. By means of coordinate transformations, the inverse kinematics of six-bar parallel mechanism is studied, and the precise equations for six bars' lengths are obtained. Based on the inverse kinematics, forward kinematics of six-bar parallel mechanism is obtained with trust region method working for nonlinear optimization. The corresponding MATLAB program is also designed. The results show that trust region method is an effective way to solve forward kinematics, and the program is stable, reliable and rapid. This method has small errors with linear precision of $10^{-12}$ mm and rotational precision of $10^{-15}$ deg. Using differential snail adjustment, monochromator chamber's attitude can reach a linear resolution of 5 μm and a rotational resolution of 3″, which entirely satisfies the practical requirements.



## 1. Introduction

Parallel mechanisms are being preferred in many applications, as they have advantages of higher stiffness, better accuracy, greater stability and more compact structure over serial mechanisms [1–3]. Beamline is an important part of synchrotron radiation facility, which is primarily made up of monochromator, focusing mirror system and so on. The key components, mirrors in these systems, are all adjusted by fine mechanisms. Wherein six-bar parallel mechanism, which has six degrees of freedom, is playing a more and more important role in the field [4–8].

In recent years, study of parallel mechanisms has been a hotspot of mechanism study in the world [9]. Referring to the kinematics analysis of parallel mechanisms, given the position and orientation of the moving platform, to determine the lengths of all bars, the whole process is called inverse kinematics. Contrarily, given the lengths of all bars, to determine the position and orientation of the moving platform, is called forward kinematics [10,11]. It is known that it's relatively easy to work out inverse kinematics of parallel mechanisms, while forward kinematics is much more complicated and is becoming a big problem all over the world [9]. Moreover, forward kinematics is the foundation for applications of parallel mechanisms [3]. To solve the problem, there are two algorithms: analytic solution and numerical method. Analytic solution can obtain all results of the equations, but it is complicated thus limiting its applications. Though numerical method depends on the iterative initial value, its mathematic models can be easily built and real solutions of all kinds of parallel mechanisms will be obtained quickly. Therefore, numerical method is widely used to solve forward kinematics problem [11].

A special six-bar parallel mechanism is being applied in the monochromator of STXM beamline, one of the first-built beamlines in Shanghai Synchrotron Radiation Facility (SSRF). Its physical and mathematic model is set up in the paper. By means of coordinate transformations, the inverse kinematics of six-bar parallel mechanism is studied, and the precise equations for six bars' lengths are obtained. Curves of six bars' lengths are represented after simulation and calculation by MATLAB. Then based on inverse kinematics, trust region method, a nonlinear programming algorithm, is used to solve the forward kinematics problem. MATLAB simulation and calculations show that the results are quite accurate and can be obtained very fast by using the trust region method, which provides important guidance in our practical work.

## 2. Inverse kinematics of six-bar parallel mechanism

Parallel mechanisms were first introduced by Stewart in the 1960 s [12]. Its original model is shown as Fig. 1 and that's the classic model of six-bar parallel mechanism. It has six degrees of

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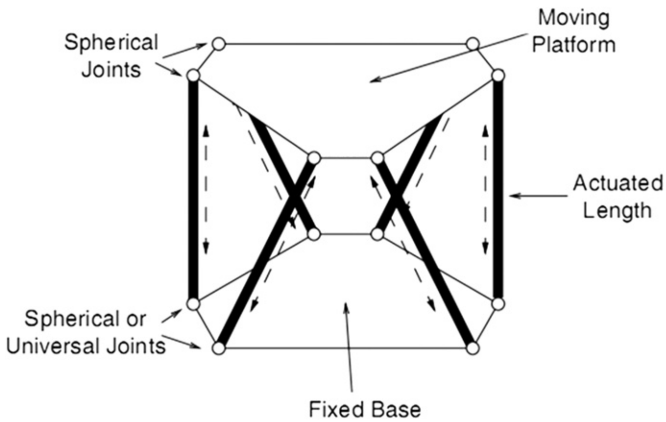


**Fig. 1.** Classic model of six-bar parallel mechanism.

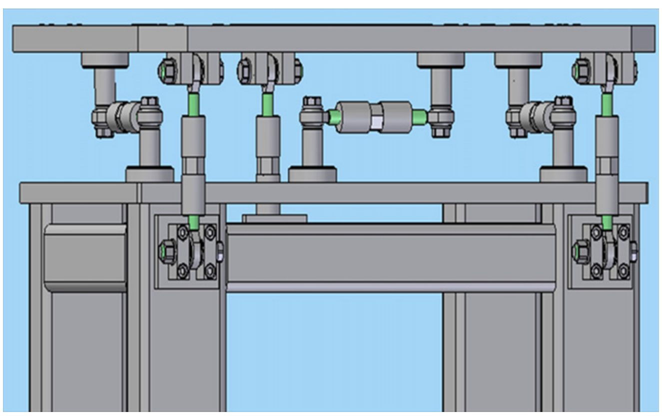

**Fig. 2.** Physical model of six-bar parallel mechanism.

freedom and its six flexible bars are connected to mobile joints at the fixed base and moving platform.

A special six-bar parallel mechanism, as shown in Fig. 2, is always applied in synchrotron radiation beamline facilities. Unlike the classic model, it has three vertical and three horizontal bars, whose lengths are fine adjusted by differential screws in particular. Upper and lower joints are not at the same plane. This mechanisms' configuration is space-saving and easy to operate.

### 2.1. Establishing coordinate systems

From the model shown in Fig. 2, the fixed base coordinate frame $O$-$xyz$ is established firstly. Set the fixed plane $Oxy$ by the lower joints of three vertical bars. Point $O$, origin of the frame, is located in the center of fixed base. Directions of coordinate axes $x$, $y$, $z$, are determined by the right-hand rule, as shown in Fig. 3. Therefore, coordinates of six lower joints in $O$-$xyz$ are $A_i=\{A_{ix}, A_{iy}, A_{iz}\}^T$ $(i=1, 2, \ldots, 6)$.

Establish the moving platform frame $O'$-$x'y'z'$. Set the moving plane $O'x'y'$ by the upper joints of three vertical bars. Actually, mirrors in synchrotron radiation beamline facilities are always higher than the moving plane, thus $O'$ is located right above plane $O'x'y'$ with the height of $h$ in the paper. Three axes of two frames are parallel to each other at the initial time. Namely, coordinates of six upper joints in $O'$-$x'y'z'$ are $B_i'=\{B_{ix}', B_{iy}', B_{iz}'\}^T$ $(i=1, 2, \ldots, 6)$.

### 2.2. Inverse kinematics

Set $(\alpha, \beta, \gamma)$ are the RPY (Roll-Pitch-Yaw) angles and $P=\{X_P, Y_P, Z_P\}^T$ is the coordinate of $O'$ in $O$-$xyz$. Coordinates of six upper joints

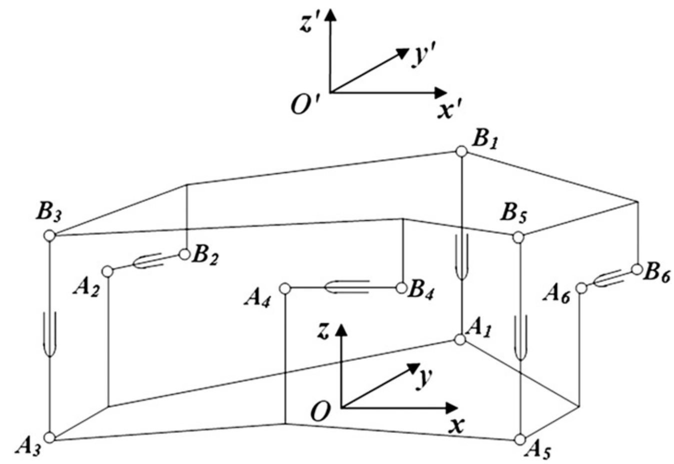


**Fig. 3.** Kinematic diagram of six-bar parallel mechanism.

in $O$-$xyz$ can be written in the following way by means of coordinate transformations

$$B_i=RB_i'+P(i=1,2,\ldots,6) \tag{1}$$

whereas $R$ is the attitude matrix from $O'$-$x'y'z'$ to $O$-$xyz$ [11,15]

$$R=Yaw(z,\gamma)Pitch(y,\beta)Roll(x,\alpha)$$
$$=\begin{bmatrix} c\gamma & -s\gamma & 0 \\ s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix} \tag{2}$$

where $c$ is on behalf of cosine rules and $s$ is sine rules.

Bars' length vectors $l_i$ in $O$-$xyz$ are

$$l_i=B_i-A_i=(B_{ix}-A_{ix})x+(B_{iy}-A_{iy})y+(B_{iz}-A_{iz})z$$
$$=l_{ix}x+l_{iy}y+l_{iz}z(i=1,2,\ldots,6) \tag{3}$$

Given the position and orientation of the moving platform, namely, $(\alpha, \beta, \gamma, X_P, Y_P, Z_P)$ are assumed to be known, lengths of six bars are as follows

$$l_i=\sqrt{(B_{ix}-A_{ix})^2+(B_{iy}-A_{iy})^2+(B_{iz}-A_{iz})^2}(i=1,2,\ldots,6) \tag{4}$$

Eq. (4) are exactly the inverse kinematic equations of six-bar parallel mechanism.

Furthermore, varied lengths of six bars are defined as

$$\Delta l_i=l_i-l(i=1,2,\ldots,6) \tag{5}$$

where $l$ is the initial length of six bars.

## 3. Forward kinematics of six-bar parallel mechanism

Forward kinematics is the process of determining the position and orientation of the moving platform, $(\alpha, \beta, \gamma, X_P, Y_P, Z_P)$, by given the lengths of six bars $l_i$ $(i=1, 2, \ldots, 6)$. From Eq. (4), constraint equations between the position and orientation of the moving platform and the bars' lengths are as follows

$$f_i(\alpha,\beta,\gamma,X_P,Y_P,Z_P,l_i)=0\ (i=1,2,\ldots,6) \tag{6}$$

Eq. (6) are nonlinear transcendental equations with six unknowns with trigonometric functions.

The key of forward kinematics is exactly to solve the nonlinear transcendental equations above. Since the emergence of parallel mechanism in 1960 s, lots of mechanists have obtained the closed-form solutions of some kinds of parallel mechanisms using analytic solutions. But due to the non-linearity of these equations, there is no complete method that would get the closed-form solution [3]. While numerical method is a sufficient way that can