JID:NUPHA AID:21173 /FLA [m1+; v1.280; Prn:27/02/2018; 15:19] P.1 (1-15)

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Nuclear Physics A ••• (••••) •••–•••

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Investigation of Bohr Hamiltonian in presence of Killingbeck potential using bi-confluent Heun functions

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Received 11 December 2017; received in revised form 20 February 2018; accepted 22 February 2018

Abstract

In this study, Bohr Hamiltonian is studied for the triaxial and rotational cases. In both cases, Killingbeck potential is used as interaction. The wave function and energy of these cases are found using bi-confluent Heun functions. The results are examined by reproducing experimental data of some isotopes for each case. Energy levels of the isotopes are shown graphically as well as theoretical results for staggering in γ bands of the isotopes is discussed. In the next step, we argue about B($E2$) transition rates of the isotopes for each case. The results have a good agreement with experimental data.

Keywords: Bohr Hamiltonian; Collective motions of atomic nuclei; Killingbeck interaction; Bi-confluent Heun functions

1. Introduction

Describing of states and properties of atomic nuclei have been one of the most important issues in theoretical nuclear physics. This topic has attracted much interests trying to obtain the closest results in comparison with experimental data to develop theoretical tools in nuclear physics [1–10]. To describe properties of atomic nuclei both, geometrical and algebraic approaches are

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https://doi.org/10.1016/j.nuclphysa.2018.02.007
0375-9474/

Please cite this article in press as: H. Sobhani et al., Investigation of Bohr Hamiltonian in presence of Killingbeck potential using bi-confluent Heun functions, Nucl. Phys. A (2018), https://doi.org/10.1016/j.nuclphysa.2018.02.007

needed. In both of them, shape changes of atomic nuclei can be described by a specific Hamiltonian. From algebraic point of view, existence of the shape phase transitions is proposed by the study of dynamical symmetries of some algebraic structures that different shapes coincide with particular dynamical symmetries [11]. From geometrical aspect, critical point symmetries can be studied by solving the collective Hamiltonian with an appropriate potential [12–14]. Investigation of eigenfunctions and eigenvalues of the Bohr Hamiltonian yields predicting the phase transitions in the shapes of the deformed nuclei [15]. Bohr Hamiltonian has been investigated in various situations by different physicist such as Fortunato [15], Bonatsos et al. [16,17], Capak et al. [18], Chabab et al. [19] and Hassanabadi et al. [20–24]. Davydov–Chaban is an approach to study collective motion of atomic nuclei. This is obtained from Bohr Hamiltonian by considering rigidity for γ in Bohr Hamiltonian. This approach also has been an important issue in nuclear physics since it covers some interesting results such as what is mentioned in articles published by Buganu and Budaca [25] or Bonatsos et al. [26]. Also, we published an article in which we discussed in detail that using Davydov–Chaban Hamiltonian how we can describe instability of unstable nuclei [27].

In this article, we are interested in analytical investigation of Bohr Hamiltonian for two well-known separable cases, the approximate and exact one, in which there is a Killingbeck interaction. So, in Sec 2, Bohr Hamiltonian for two cases and the potential are introduced. In Sec. 3, experimental data related to energy and $B(E2)$ transition rates of some isotopes are reproduced.

2. Bohr Hamiltonian and Killingbeck interaction

The original Bohr Hamiltonian [28,40] is

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin(3\gamma)}\frac{\partial}{\partial\gamma}\sin(3\gamma)\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_{k=1}^{3}\frac{Q_k^2}{\sin^2\left(\gamma - \frac{2}{3}\pi k\right)}\right] + V(\beta,\gamma) \tag{2.1}$$

in which, intrinsic deformation coordinates are denoted by β (deformation coordinate measuring departure from the spherical shape), γ (the angle measuring departure from axial symmetry) and Q_k $(k = 1, 2, 3)$ the operators of the total angular momentum projections in the intrinsic reference, mass parameter and Euler angles are shown by B and θ respectively. Considering the wave function as $\Psi^{as(es)}(\beta,\gamma,\theta_i) = \xi^{as(es)}_{L,\alpha}(\beta)\,\Gamma^{as(es)}(\gamma)\,D^L_{M,\alpha}(\theta_i)$ where $D^L_{M,\alpha}(\theta_i)$ represents Wigner functions, M and α are the eigenvalues of the projections of angular momentum on the laboratory fixed $\hat{z}$ axis and the body-fixed $\hat{x}'$ axis respectively, *es* stands for exact separation and *as* means approximate separation case, we want to study the Bohr Hamiltonian for two cases.

2.1. Triaxial case with $\gamma \approx \frac{\pi}{6}$

In this case in which the potential has a minimum around $\gamma \approx \frac{\pi}{6}$ one can write the last term of Eq. (2.1) in the form [40]

$$\sum_{k=1}^{3}\frac{Q_k^2}{\sin^2\left(\gamma - \frac{2\pi}{3}k\right)} \approx Q_1^2 + 4\left(Q_2^2 + Q_3^2\right) = 4\left(Q_1^2 + Q_2^2 + Q_3^2\right) - 3Q_1^2. \tag{2.2}$$