Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics A 957 (2017) 406–415

NUCLEAR PHYSICS A

www.elsevier.com/locate/nuclphysa

Classical electromagnetic fields from quantum sources in heavy-ion collisions

Robert Holliday, Ryan McCarty, Balthazar Peroutka, Kirill Tuchin [*]

Department of Physics and Astronomy, Iowa State University, Ames, IA, 50011, USA

Received 5 May 2016; received in revised form 19 September 2016; accepted 13 October 2016

Abstract

Electromagnetic fields are generated in high energy nuclear collisions by spectator valence protons. These fields are traditionally computed by integrating the Maxwell equations with point sources. One might expect that such an approach is valid at distances much larger than the proton size and thus such a classical approach should work well for almost the entire interaction region in the case of heavy nuclei. We argue that, in fact, the contrary is true: due to the quantum diffusion of the proton wave function, the classical approximation breaks down at distances of the order of the system size. We compute the electromagnetic field created by a charged particle described initially as a Gaussian wave packet of width 1 fm and evolving in vacuum according to the Klein–Gordon equation. We completely neglect the medium effects. We show that the dynamics, magnitude and even sign of the electromagnetic field created by classical and quantum sources are different.

Keywords: Electromagnetic field; Heavy-ion collisions

---

1. Introduction

Highly intense electromagnetic fields are created in relativistic nuclear collisions. The first possible phenomenological manifestation of these fields in proton-proton collisions was pro-

---

[*] Corresponding author.
E-mail address: tuchink@gmail.com (K. Tuchin).

http://dx.doi.org/10.1016/j.nuclphysa.2016.10.003

posed in 1988 by Ambjorn and Olesen [1], who suggested that they may cause condensation of W-bosons. Recently, it has been realized that the electromagnetic fields may get frozen into the Quark Gluon Plasma produced in these relativistic heavy-ion collisions [2], which has numerous phenomenological consequences [3,4]. How strong the effect of the electromagnetic field is on observable quantities depends on the strength of the field at the time of collision and on its subsequent time-evolution. This information is encoded in Maxwell's equations, which can be solved for a given distribution of electromagnetic currents.

All calculations of the electromagnetic fields thus far assumed that the sources are electrically charged point particles counter-propagating along the straight lines at a distance b away from each other [5–11]. The resulting field strength is inversely proportional to the square of the impact parameter b. At small b the classical approximation breaks down, which is manifested by the divergence of the field strength. One can readily fix this problem by imposing an ultra-violet cutoff, thereby effectively converting the point charges into hard spheres [7,10]. However, this prescription leads to a significant variation in field strength estimates. Since the largest contribution to the electromagnetic fields arises at short distances, where the classical approximation breaks down, a consistent calculation requires the full quantum treatment of the electromagnetic currents. Such a treatment constitutes the main subject of the present article. The classical approximation is expected to break down at distances $b \sim 1/m$, because a particle's position cannot be localized better than its Compton wave length. In fact, we argue that this happens at much larger distances.

In the quantum approach, the colliding nucleons are described by a wave function satisfying the Dirac equation. Since the charge distribution in quantum treatment is not singular, the strength of the electromagnetic field naturally saturates in the UV limit. As in the classical case, we employ the eikonal approximation [5–11], assuming that nucleons move along the straight lines thus neglecting their deflection due to the external fields. This approximation breaks down at very large momentum transfer corresponding to very small $b \sim \alpha_s/m\gamma$, where m is the nucleon mass and γ is the collisions energy in units of m. The principal practical difference between the quantum and classical approaches is the quantum diffusion effect of the nucleon wave function. In contrast to the classical picture where the charge moves as a whole at nearly the speed of light, quantum diffusion generates a sideway flow of current. As a consequence, the electromagnetic field stays longer in the interaction region. This effect does not require any medium and is present even in vacuum. Therefore, in this article we focus on the electromagnetic fields created by the quantum sources in vacuum. Also, since quantum diffusion does not depend on spin, we replace the nucleons with scalar sources satisfying the Klein–Gordon equation. We defer analysis of the spin contribution and medium effects to future work.

It is important to emphasize, that unlike the typical situation in scattering theory, we cannot approximate the nucleon wave function by a plane wave, i.e. a state with a definite momentum. The collision theory usually considers scattering of two wave packets that have a spatial extent much greater than the interaction range because the former is of macroscopic origin, while the latter is of a microscopic scale. This is one of the reasons that the incident and outgoing particles can be treated as plane waves [12]. Our situation is different because both scales are microscopic. The very fact that a collision at impact parameter b happened implies that the spatial dimension of the wave packet after the collision is of order b, which is comparable with the interaction range. If we were interested in the short distance processes, we could have used the plane approximation because such a process probes only a small part of the wave function in momentum space. Since we are interested in the long-range electromagnetic fields, the plane wave approximation cannot be made.