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New numerical method for fission half-lives of heavy and superheavy nuclei at ground and excited states

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Abstract

The spontaneous fission half-lives for heavy and superheavy nuclei between U and Hs isotopes are calculated in framework of the generalized liquid drop model by applying a new method of numerically solving Schrödinger equation compared with the semi-empirical WKB approximation. The calculated half-lives are in very good agreement with the experimental data, indicating the reliability of the new approach. The second part of this work is to estimate the fission half-lives of ^{238}Np* at excited state of 7.3 MeV and ^{239}U* at excited states of 7.081, 8.078, 8.387 and 8.989 MeV with the numerical method. The estimated results compared with the experimental values and with the results by WKB approximation show the numerical method is applicable to both the spontaneous fission and excited fission.

Keywords: Spontaneous fission; Excited fission; Numerical method; WKB approximation

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1. Introduction

Spontaneous fission is one of the most prominent decay modes, energetically feasible for heavy and superheavy nuclei (SHN). Early descriptions of fission were based on a purely geometrical framework of the charged liquid drop model [1]. In order to quantify just how much the minimum energy should be required to cause a given nucleus fission, Bohr and Wheeler

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introduced the concept of fission barrier. They assumed that with a set of deformation parameters spanning a deformation space, the geometry shapes of a mother nucleus can be described along the process of fission. The potential surface as well as the fission barrier then can be estimated as well. In 1955, Swiatecki [2] suggested that more realistic fission barriers could be obtained by adding a correction energy to the minimum in the liquid drop model barrier as a fact that there exists non-ignorable difference between the experimentally observed nuclear ground-state mass and that given by the liquid drop model. Several year later, Strutinsky [3] put forward a approach of adding the quantum shell effects to the average behavior described by the liquid drop, to theoretically deal with the problem, which turned out to be very successful in explaining many features of spontaneous fission [4–6].

In this work, the fission barriers are obtained within a Generalized Liquid Drop Model (GLDM), which taking into account the proximity energy between close opposite surfaces, the mass and charge asymmetry, the phenomenological pairing correction and the microscopic shell correction. The proximity forces strongly lower the deformation energy of compact and creviced shapes. As a result, the calculated potential barrier heights are well in agreement with the experimental results [7,8]. Within the same approach, the α-decay [9–12], cluster emission [13] and fusion [14] data are also reproduced. Recently, the spontaneous fission half-lives of several SHN have been measured by different laboratories [15–18]. The fission as well as α-decay probability determines the stability of these newly synthesized SHN. In the view of classical mechanics, fission occurs only when the release energy is higher than the barrier, while in theory of quantum mechanics there is still some probability to penetrate the barrier even with energies below the fission barrier. The WKB framework is widely performed for the α-decay along with other physically analogical decay processes in calculating the quantum mechanical tunneling probability. However, it should be noted that the WKB approximation works well only at energies well below the barrier [19]. In fact, the accuracy of the WKB approximation also depends on the shape of the potential barrier as well as the decay energy. So, it may be safer to adopt the numerical method to directly solve the one-dimensional Schrödinger equation. The present work is focused on the comparison between the numerical approach and the WKB approximation in cases of spontaneous fission and excited fission. However, the excited fission is much more complex than the spontaneous fission and it has been rarely theoretically investigated in the past. In order to simplify the calculations, this work for the first attempt applied the GLDM to deal with the excited fission since it has made perfect calculations for the half-lives in spontaneous fission. We build the excited fission barrier with GLDM by some simple modifications due to excited nuclear structure. The barrier penetration probabilities are calculated with both the numerical method and WKB approximation. But, even so, we cannot perform numerous calculations because the experimental fission lifetime of heavy and superheavy nuclei at low excited state is so rarely published. This work only estimate the fission lifetimes of ^{238}Np* at excited state of 7.3 MeV and ^{239}U* at excited states of 7.081, 8.078, 8.387 and 8.989 MeV since their experimental average lifetimes have been measured (Ref. [20] for ^{238}Np* and Refs. [21,22] for ^{239}U*).

The paper is organized as follows. The theory of generalized liquid drop model is briefly reviewed in Section 2. In Section 3, there are two methods of barrier penetration probability calculations presented. Results and discussions are given in Section 4, and conclusions and summary are provided in Section 5.