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Fluctuation induced equality of multi-particle eccentricities for four or more particles

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Abstract

We discuss eccentricities (ellipticity and triangularity) generated in nucleus–nucleus and proton–nucleus collisions. We define multi-particle eccentricities $\epsilon_n\{m\}$ which are associated with the n'th angular multipole moment for m particles. We show that in the limit of fluctuation dominance all of the $\epsilon_n\{m\}$'s are approximately equal for $m \geqslant 4$. For dynamics linearly responding to these eccentricities such as hydrodynamics, these relations among eccentricities are translated into relations among flow moments $v_n\{m\}$. We explicitly demonstrate it with hydrodynamic calculations.

Keywords: p + A collisions; Azimuthal anisotropy; Ellipticity; Glauber model; Hydrodynamics

1. Introduction

In nucleus–nucleus (A + A) and proton–nucleus (p + A) collisions, there is an approximately boost invariant structure associated with an angular asymmetry of the two and many particle

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correlation functions. In heavy ion collisions, this asymmetry is conventionally associated with hydrodynamic flow driven by angular asymmetries of the underlying matter distribution [1]. This angular asymmetry contains a component that is due to fluctuations in the transverse positions of particle interactions [2,3], and a component associated with source asymmetry at finite impact parameter of the collision. In p + A collisions, a variety of mechanisms leading to angular correlations have been proposed, some involving hydrodynamic like scenarios as in A + A interactions [4] and some involving non-trivial angular correlations associated with the emission process [5].

To quantify the momentum space distribution of particles, one may identify an angular harmonic of the momentum space distribution as

$$v_n = \frac{1}{N} \int d^2 p_T \, e^{in\phi} \frac{dN}{dy \, d^2 p_T} \tag{1}$$

where $N = \int d^2 p_T \frac{dN}{dy\, d^2 p_T}$. This is a complex quantity and for each event is of the form

$$v_n = \eta_n e^{i\gamma_n} \tag{2}$$

The angle γ_n describes the orientation of the flow vector relative to some chosen coordinate axis, and η_n is its modulus. When averaging over events, it must be true that $\langle v_n \rangle = 0$ by rotational invariance.

Borghini, Dinh and Ollitrault introduced multi-particle correlations that measure the rotationally invariant part of the flow [6]. For example, the two particle correlation is

$$v_n^2\{2\} = \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle = \left\langle |v_n|^2 \right\rangle \tag{3}$$

In a collision at fixed impact parameter, it is conventionally believed that this expectation value contains a piece associated with the geometry of the collision, which if the impact parameter is sufficiently well defined is non-fluctuating, plus a fluctuating component. Originally these authors introduced higher order components associated with 4, 6 and more particles correlations to reduce contributions from two (four or more) particles non-flow correlations. For example

$$v_n^4\{4\} = 2\left\langle |v_n|^2 \right\rangle^2 - \left\langle |v_n|^4 \right\rangle \tag{4}$$

and

$$v_n^6\{6\} = \frac{1}{4}\left[\left\langle |v_n|^6 \right\rangle - 9\left\langle |v_n|^2 \right\rangle \left\langle |v_n|^4 \right\rangle + 12\left\langle |v_n|^2 \right\rangle^3\right] \tag{5}$$

and higher order generalizations to larger number of particles. Interesting for the following discussion is the fact, that the cumulant expressions for the harmonic flow coefficients allow one to isolate the fluctuating component from that of the geometry of the collision. If there is no mean field contribution to the distribution functions then, it is easy to see that for purely Gaussian fluctuations, $v_n\{4\} = v_n\{6\} = 0$, that is these terms are sensitive only to correlations of fourth order or higher from the average flow [7]. For a large number of independent sources, these correlations are expected to be small. So it is believed that these higher order correlations capture the intrinsic flow contributions and reduce the effects of fluctuations.

In heavy ion collisions it is in fact found that with very good precision $v_2\{4\} = v_2\{6\} = v_2\{8\}$, with $v_2\{2\}$ different due to its intrinsic fluctuations [8,9]. The pattern seen so far in p + A collisions is remarkably similar to that found in heavy ion collisions [10,11]. Why is this result surprising? It is because if one measures the magnitude of flow fluctuation by $\sqrt{\frac{v_2\{2\}^2 - v_2\{4\}^2}{v_2\{2\}^2 + v_2\{4\}^2}}$, one