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Inflection point as a characteristic of the QCD critical point

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Abstract

In the universality class of the critical point of QCD, the appearance of the inflection point in the equation of state (EoS) is associated with the second order phase transition. The high cumulants of conserved quantities near the critical point are corresponding to the high derivatives of the EoS near the inflection point. The critical behavior of high cumulants of conserved charge near the QCD critical point, in particular, the sign change, is closely related to the appearance of inflection point. We show in general how the times of sign change of high cumulants relate to the order of derivative. We also demonstrate that the character of inflection point of EoS is as visible as the sign change of high cumulants in 3 systems, i.e., the van der Waals equation of fluid, the energy of spin model and the baryon number density of QCD matter. Therefore, we propose that the EoS, or the mean of baryon number density, should be measured and studied together with its higher cumulants in exploring the QCD critical point in heavy ion collisions.

Keywords: Heavy ion collisions; Quark deconfinement; QCD critical point; Inflection point; High cumulant

1. Introduction

The theory of strong interaction—quantum chromodynamics (QCD)—has a complicated phase structure. Mapping the QCD phase diagram in the temperature (T) and baryon chemical potential (μ_B) plane is currently one of the main goals of high energy nuclear physics. Lat-

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tice QCD calculations indicate that the chiral and deconfinement phase transitions are a smooth crossover at zero baryon chemical potential [1], while several QCD-based models predict a first order phase transition at high density [2–6]. The existence of the QCD critical point (CP), which terminates the first order phase transition line in the QCD phase diagram, is expected and being searched for in the ongoing heavy ion experiments [7].

Locating the QCD critical point is a challenge for both the theorists and the experimentalists. Due to that the precise position of the critical point is not well known from the theoretical side, a measurement from the experimental side is highly needed. By varying the center-of-mass energy $\sqrt{s_{\rm NN}}$ of the nucleus–nucleus collisions, we can scan large regions of the phase diagram. The chemical freeze-out at different $\sqrt{s_{\rm NN}}$ happens at different positions along the freeze-out curve [8] and therefore the trajectories of the reaction systems in the T–$\mu_{\rm B}$ plane have to cross different areas of the phase diagram and might even hit the critical area. When the trajectories of the reaction systems in the T–$\mu_{\rm B}$ plane get close to the critical point, large fluctuations appear. Assuming freeze-out happens near the CP, the large fluctuations can survive and should be observed. A non-monotonic behavior of the fluctuations from low energy to high energy is expected [9]. Several event-by-event fluctuation observables, e.g. mean transverse momentum fluctuations and the multiplicity fluctuations (here the fluctuation measures relate to the variances of the event-variable distributions, or the second order cumulants), have been analyzed at the SPS energies with $5 < \sqrt{s_{\rm NN}} < 17$ GeV [10] and at the RHIC energies with $7 < \sqrt{s_{\rm NN}} < 200$ GeV [11]. NA49 at SPS finds peaks in the system size dependence of the mean transverse momentum fluctuation and the multiplicity fluctuation, which agrees with predictions for the CP. However, the energy dependences of the two measures do not show any signatures for CP. The evidences for a critical point are inconclusive so far [12].

The critical point means the divergent correlation length in the thermodynamic limit. While in a realistic heavy ion collision the correlation length ξ gets at most 2–3 fm because of the finite system size and the limited evolution time [13]. Since higher cumulants are proportional to a higher power of ξ, higher cumulants of conserved charges are more sensitive to the critical point and were proposed as a promising observable for the search of the QCD critical point [14–16]. The cumulants are calculated from corresponding probability distributions in experiments. Theoretically, according to the grand canonical formulation of thermodynamics, the cumulants for charge X are proportional to generalized susceptibilities, which are derivatives of the pressure with respect to the corresponding Lagrange multiplier $\mu_{\rm X}$, i.e.

$$\chi_n^{\rm X} = \frac{\partial^n \widehat{p}}{\partial \widehat{\mu}_{\rm X}^n}, \tag{1}$$

where $\widehat{\mu}_{\rm X} = \mu_{\rm X}/T$ and $\widehat{p} = P/T^4$. Since the baryon number density is $\langle n_{\rm B}\rangle = \frac{\partial P}{\partial \mu_{\rm B}}$, the nth order cumulant is related to the $(n-1)$th order derivative of $\langle n_{\rm B}\rangle$ with respect to $\mu_{\rm B}$, i.e.

$$\chi_n^{\rm B} \sim \frac{\partial^{n-1}\langle n_{\rm B}\rangle}{\partial \mu_{\rm B}^{n-1}}. \tag{2}$$

Besides the cumulants of charges, the cumulants of the energy are also proposed as probes of the QCD phase transition [17,18]. At $\mu = 0$, the cumulants of the energy measure the derivatives of the EoS [18], i.e.

$$\langle(\delta E)^n\rangle = \left(-\frac{\partial}{\partial 1/T}\right)^{n-1}\langle E\rangle. \tag{3}$$