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Layered plate with discontinuous connection: Exact mathematical model

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ABSTRACT

The subject of this paper is the plate composed of two identical layers connected to each other in a discontinuous way, i.e. via discontinuous elements (connectors). This paper presents a model that describes the mechanical behavior of this plate by a system of exact, analytical (explicit) equations. The discrete distribution of discontinuous connectors is replaced by a fictitious continuous medium (interlayer). Accordingly, the plate is modeled as an equivalent three-layered plate: Two outer layers and a connecting inner interlayer. In order to obtain a fast and easy to use tool, something that is necessary for an analytical model to be chosen over finite elements and empirical formulas, modeling process is developed within the framework of two-dimensional elasticity. In so doing, the model also represents a means for attaining full comprehension of the mechanical phenomena that are involved, something that neither three-dimensional elasticity nor finite elements and empirical formulas can attain. The transition from three to two-dimensional behavior is obtained by relating the normal stress in the direction transverse to the plate to the distortion in the interlayer. The two-dimensional behavior is governed using kinematic and force assumptions that do not impose appreciable constraints on the stress–strain state and structural behavior. Starting from these assumptions, the paper develops the relationships between displacements and interface stresses, for both continuous and discontinuous connection. The latter relationships, which are used in this model, and the former relationships, which were used in a previously presented model, are discussed and compared to each other. The subsequent sections of the paper describe the model and present some real case applications of discontinuously-connected layered plate.

1. Introduction

This paper represents the fifth step of a research line whose scope is the analytical and exact modeling of composite flexural members. The first two steps were devoted to the composite beam [1,2]; the third and fourth steps were devoted to the composite plate with continuous connection [3,4]. This last step is devoted to the composite plate with discontinuous connection.

A composite structure can be defined as a structure made up of distinct components connected to each other. The connection transfers in-plane shear stresses between the components, which provide the composite structure with greater stiffness and strength than the sum of its components as standalone structural elements. Composite structures can be broadly subdivided into mixed, laminated, sandwich, and layered structures.

Mixed (or hybrid) structures are composite structures whose components are made of materials different from one another. Mixed structures use the in-plane shear stress transfer to make the components behave in the most advantageous way. Examples are steel–concrete beams [5–7], which make the concrete carry prevailing compressive stress and the steel prevailing tensile stress, and timber–concrete structures [8], which make the concrete carry prevailing compressive stress. The connection between the components is normally obtained by using discontinuous elements, named shear connectors. Examples of shear connectors are welded (headed) studs, short length of steel channels, bolts, series of sharp teeth (which dig into the wood), connection enhancements (ribs, embossments, indentations), welded steel (spiral) bars.

Laminated structures are composite structures manufactured by connecting two or more layers together, with thin interlayers. Lamination allows the composite structure to tolerate impacts and concentrated loads, and to dissipate the energy released by cracking, so attenuating crack propagation. Moreover, a laminated structure may have non-structural advantages as regards thermal insulation, sound attenuation, and radiation absorption. On the other hand, lamination implies a certain reduction in stiffness and strength, with respect to the equivalent monolithic structure (i.e., the monolith with thickness equal to the total thickness of the laminated). Thus, laminated structures use the interface in-plane shear stresses to minimize those reductions. Examples are polymeric laminates [9] and laminated glass [10–16]. The polymeric interlayer of laminated glass, in particular, prevents the

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Nomenclature

B	side of the layered plate parallel to y-axis
E	elastic modulus of the layers (in tension and compression)
E_s	elastic modulus of the connector (discontinuous connecting element)
E_t	elastic modulus of the interlayer, which is assumed to be nil
G	shear elastic modulus of the layers
G_t	shear elastic modulus of the interlayer
I_s	modulus of inertia of the individual connector (discontinuous connecting element)
h	thickness of each layer
L	side of the layered plate parallel to x-axis, with $L \geqslant B$
λ	half the thickness of the interlayer
M	$m \cdot \pi / L$
N	$n \cdot \pi / B$
p	lateral load, applied on the upper surface of the upper layer (F/L^2)
t_u, t_v	x and y components of the in-plane shear stress transmitted through the upper interface, between the interlayer and the upper layer
u, v, w	components of displacements in the x, y, z directions, respectively
u_t, v_t	x and y components of the relative in-plane displacement of the upper interface with respect to the to the plate middle surface (which is immovable)
t_{vm}, v_{tm}	maximum value of t_v and v_t
T_u	shear force transmitted by an individual connecting element in the x direction
w_{Max}	maximum deflection of the plate (at the "center")
z', z''	out of plane axes with origin O' and O'' on the middle plane of the upper and lower layers, respectively
Δ_x, Δ_y	x-axis and y-axis spacing of the connectors
ν	Poisson's ratio of the layers
σ_{yi}	σ_y stresses at the upper surface of the lower layer
σ_{ym}	σ_y stresses at the lower surface of the lower layer
σ_{Max}	maximum stress in the layered plate
σ_z	normal stress in the direction transverse to the plate
θ_{ab}	angle of rotation of the segment ab
ζ	$h + 2 \cdot \lambda$ = lever arm of the internal couple of the in-plane forces

propagation of a crack to the other glass ply, eliminates falling glass shards, and guarantees substantial residual strength after cracking [16].

Sandwich structures are composite structures fabricated by attaching two relatively stiff skins (facings or face-sheets) to a low-to-moderate stiffness core. The design criterion of the cross-section is to place a stiff material where the normal stresses and their bending lever arms are greater, i.e. outward from the centroid, and a soft material where the normal stresses and their bending lever arms are lower, i.e. inward to the centroid. In other words, an interlayer is interposed between two layers with the purpose of increasing both the resultant forces in each layer and their lever arm, without increasing the maximum stress acting in the layers. Considering that the soft material is much lighter than the stiff material and that the stiff material has also adequate strength, sandwich structures can achieve higher stiffness-to-density and strength-to-density ratios than other comparable structures, although the stiffness of the interlayer may be drastically lower than that of a layer. Examples of skin materials are reinforced polymers, metals, and timber [17–25]. The core can be composed of a continuous material—examples are polymers, foams, balsa wood [17,18,20–25]—or a micro-structured periodic material—examples are honeycombs, corrugated, lattice, ceramic tiles [19,26–29].

Layered structures are composite structures manufactured by connecting two or more equal layers together, without interposing any space interlayer. The connection can be either discontinuous or continuous. Examples of discontinuous connections are studs, nails, dowels, pins, spikes, screws, dovetails, comb joints, fingers [30–44]. Examples of continuous connections are polymers (chemical bonding), welds, post-vulcanization, interlocking devices (cohesion and mobilized friction mechanisms between the rough interfaces) [9,18–20,22,31–33]. The difference between a layered and a mixed structure is that the components of the latter are different from one another, contrary to the former. The difference between a layered and a laminated structure is that the latter is assembled together through the interposition of one or more space layers (thin, but not negligible), contrary to the former. The same is true for a sandwich structure, with the difference that the interlayer may be thick.

This paper focuses on the layered plate with discontinuous connection (two layers). Laminated plates with discontinuous interlayer [5–8,16] and sandwich plates with discontinuous core are included [17–19,25,34,45]. Moreover, this research is intimately connected with the topic of composite structures with imperfect interface, which provides important Refs. [46–54].

2. Standard modeling assumptions

Analytical modeling of composite plates can be developed in the framework of either three-dimensional or two-dimensional elasticity.

Exact three-dimensional elasticity solutions were constructed in 1969 for the composite laminates in cylindrical bending [55] and in 1970 for rectangular bidirectional composites and sandwich plates [56]. These analytical models represent important achievements not only in sandwich theory but in mechanics. Unfortunately, these models are neither easy nor expressive, and it is neither straightforward nor fast to obtain the solutions, something that conversely is necessary for an analytical model to be chosen over finite element models or empirical formulas. Moreover, the three-dimensional approach does not capture the defining attribute of layered structures, because it does not distinguish between essential behaviors and side effects. Thus, three-dimensional analytical modeling does not even represent a means for attaining scientific understanding of the mechanical behavior of the layered plate.

In the two-dimensional elasticity framework, conversely, modeling process can be developed so as to obtain a simple analytical formulation whose application not only is very easy, but even drastically less time consuming than to generate any finite element mesh or to apply any empirical formula [3,4]. Furthermore, the two-dimensional approach reproduces directly and explicitly the process of deformation and mechanical failure of the layered plate and the connection between these processes and their underlying mechanisms, which provides fully understanding of the phenomena involved. For these reasons, this research was developed within the framework of the two-dimensional elasticity.

Two-dimensional elasticity differs from three-dimensional elasticity mainly in the normal stress in the direction transverse to the