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Initial state and thermalization

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Abstract

We report recent results on the role of instabilities in the isotropization and thermalization of a longitudinally expanding system of quantum fields.

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1. Introduction

The evolution of the quark-gluon matter produced in ultrarelativistic heavy ion collisions is apparently well described by hydrodynamics with a very small viscosity. Given its derivation from an expansion of the energy-momentum tensor in gradients of the velocity field, one expects hydrodynamics to be applicable only when the transverse and longitudinal pressures are sufficiently close, and if the system is close enough to local thermal equilibrium.

An outstanding problem in the application of Quantum ChromoDynamics to the description of the early stages of these collisions is to justify that these conditions are indeed met at a short enough time to apply hydrodynamics. The main QCD tool for performing these studies is the Color Glass Condensate (CGC) framework [1], an effective theory designed to describe the highly occupied gluon states encountered in the wave functions of high energy nuclei, and their interactions in a collision. In the CGC, the fast partons ($k^+ > \Lambda^+$ for a nucleus moving in the $+z$ direction) are described as static (thanks to time dilation of all the internal dynamics in a high energy nucleus) color sources $J^\mu \sim \delta^{\mu +}\rho$ on the light-cone, with a probability $W[\rho]$. The gluons below the cutoff ($k^+ < \Lambda^+$) are described as ordinary gauge fields, coupled eikonally to the fast sources. When this effective description is applied to a collision, the current J^μ is the sum of two terms, corresponding respectively to each of the colliding nuclei.

2. Initial state factorization

An important issue in applications of the CGC to the calculation of observable quantities is the dependence on the cutoffs that are introduced to separate the fast and slow partons. This cutoff is not a physical quantity and should cancel in observables. However, since it is the upper bound in loop integrals, higher order corrections usually produce logarithms of the cutoff. It is well known that in electron–nucleus collisions, these logarithms can be absorbed by letting the distribution $W[\rho]$ depend on the scale Λ^+ according to the JIMWLK equation [2]. For the CGC to provide a consistent description of nucleus–nucleus collisions, one needs to prove that the same distributions $W[\rho]$'s are

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sufficient to absorb all the logarithms that arise in these collisions, and that this works for a sufficiently large class of observables.

The first step in this proof is to show that at Leading Order (LO) in the strong coupling α_s, all the inclusive observables (i.e. those that do not restrict the content of the final state, such as particle spectra, or the expectation value of some local operators in the final state) can be expressed in terms of the retarded solution of the classical Yang-Mills equations that vanishes in the remote past ($x^0 \to -\infty$) [3]. For instance, in terms of this classical field $\mathcal{A}^\mu$, the gluon spectra at LO read

$$\left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO} \sim \int d^4x d^4y\; e^{ip\cdot(x-y)}\; \Box_x\Box_y\; \mathcal{A}(x)\,\mathcal{A}(y)\;, \qquad \left.\frac{dN_n}{d^3\vec{\boldsymbol{p}}_1\cdots d^3\vec{\boldsymbol{p}}_n}\right|_{\rm LO} = \left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}_1}\right|_{\rm LO}\cdots\left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}_n}\right|_{\rm LO}\;. \tag{1}$$

The next step is to compute the Next to Leading Order (NLO) correction to these observables. In [4], we have shown that for an inclusive observable O, the LO and NLO contributions are formally related by

$$O_{\rm NLO} = \left[\frac{1}{2}\int_{\boldsymbol{u},\boldsymbol{v}}\int_{\boldsymbol{k}} [\boldsymbol{a_k}\,\mathbb{T}]_{\boldsymbol{u}} [\boldsymbol{a}^*_{\boldsymbol{k}}\,\mathbb{T}]_{\boldsymbol{v}} + \int_{\boldsymbol{u}} [\boldsymbol{\alpha}\,\mathbb{T}]_{\boldsymbol{u}}\right] O_{\rm LO}\;. \tag{2}$$

In this formula, $\mathbb{T}$ is the operator that generates shifts of the classical field at some initial time (and the integrals over $\boldsymbol{u},\boldsymbol{v}$ are integrals over the spatial coordinates at this time). The quantities $a_{\boldsymbol{k}}, \alpha$ are small perturbations to the classical field, evaluated at the same time. The formula is true for any initial time, but some choices simplify the problem by making $a_{\boldsymbol{k}}, \alpha$ calculable analytically. Then, the main result in proving the factorization of the logarithms of the cutoffs is

$$\frac{1}{2}\int_{\boldsymbol{u},\boldsymbol{v}}\int_{\boldsymbol{k}} [\boldsymbol{a_k}\,\mathbb{T}]_{\boldsymbol{u}} [\boldsymbol{a}^*_{\boldsymbol{k}}\,\mathbb{T}]_{\boldsymbol{v}} + \int_{\boldsymbol{u}} [\boldsymbol{\alpha}\,\mathbb{T}]_{\boldsymbol{u}} = \log(\Lambda^+)\;\mathcal{H}_1 + \log(\Lambda^-)\;\mathcal{H}_2 + \text{terms w/o logs}\;, \tag{3}$$

where $\mathcal{H}_{1,2}$ are the JIMWLK Hamiltonians of the two nuclei. This formula shows that the logarithms of the two cutoffs do not mix, since they are multiplied by operators that act on a single nucleus. From this formula, it is easy to show that one can absorb all the leading logarithms in two distributions $W_{1,2}$ that evolve with their respective JIMWLK equation,

$$\langle O\rangle_{\rm Leading\ Log} = \int [D\rho_1\, D\rho_2]\; W_1[\rho_1]\; W_2[\rho_2]\; O_{\rm LO}[\rho_1,\rho_2]\;. \tag{4}$$

It is the universality of the distributions W (i.e. the fact that the same distributions enter in different types of reactions involving the same nuclei) that gives its predictive power to the CGC.

3. Thermalization and isotropization

3.1. Resummation of the unstable modes

At LO, the classical color field produced in a nucleus-nucleus collisions corresponds to longitudinal chromo-electric and chromo-magnetic fields (figure 1 - left) that form *flux tubes* whose transverse size is of the order of the inverse saturation momentum. The energy momentum tensor of this field configuration has a negative longitudinal pressure, and is thus quite far from what can be reasonably described by hydrodynamics.

Moreover, it has been known for some time that the classical solutions of Yang-Mills equations have instabilities: small rapidity dependent perturbations to the boost invariant classical field encountered at LO grow exponentially in time, and eventually become as large as the classical field itself [5]. Such instabilities occur in loop corrections and lead to secular divergences in observables. These unstable contributions can be tracked by an improvement of the power counting, where one assigns an exponential of time to each operator $\mathbb{T}$. Then one sees that the terms with the strongest divergences (one exponential factor per power of the coupling g – see figure 1, right, for examples of 2-loop graphs that differ in the strength of their secular divergences) are obtained by exponentiating the operator in eq. (2). This exponentiation performs a resummation of a selected subset of all the higher loop corrections. When applied to