Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 933 (2018) 65–133

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

The geometry of gauged linear sigma model correlation functions

Andreas Gerhardus, Hans Jockers *, Urmi Ninad

Bethe Center for Theoretical Physics, Physikalisches Institut der Universität Bonn, Nussallee 12, D-53115 Bonn, Germany

Received 9 May 2018; accepted 13 June 2018
Available online 15 June 2018
Editor: Stephan Stieberger

Abstract

Applying advances in exact computations of supersymmetric gauge theories, we study the structure of correlation functions in two-dimensional $\mathcal{N} = (2, 2)$ Abelian and non-Abelian gauge theories. We determine universal relations among correlation functions, which yield differential equations governing the dependence of the gauge theory ground state on the Fayet–Iliopoulos parameters of the gauge theory. For gauge theories with a non-trivial infrared $\mathcal{N} = (2, 2)$ superconformal fixed point, these differential equations become the Picard–Fuchs operators governing the moduli-dependent vacuum ground state in a Hilbert space interpretation. For gauge theories with geometric target spaces, a quadratic expression in the Givental I-function generates the analyzed correlators. This gives a geometric interpretation for the correlators, their relations, and the differential equations. For classes of Calabi–Yau target spaces, such as threefolds with up to two Kähler moduli and fourfolds with a single Kähler modulus, we give general and universally applicable expressions for Picard–Fuchs operators in terms of correlators. We illustrate our results with representative examples of two-dimensional $\mathcal{N} = (2, 2)$ gauge theories.

* Corresponding author.
E-mail addresses: gerhardus@th.physik.uni-bonn.de (A. Gerhardus), jockers@uni-bonn.de (H. Jockers), urmi@th.physik.uni-bonn.de (U. Ninad).

https://doi.org/10.1016/j.nuclphysb.2018.06.008
0550-3213/

1. Introduction

With the seminal work [1] on two-dimensional $\mathcal{N} = (2, 2)$ supersymmetric gauged linear sigma models, Witten offered a powerful machinery to study the geometry of the gauge theory target spaces together with their moduli spaces in terms of gauge theory techniques. For instance, Morrison and Plesser computed quantum–exact correlation functions as functions of the Fayet–Iliopoulos parameters and the theta angles in such gauged linear sigma models [2]. Geometrically, such correlators become sections on the quantum Kähler moduli space of the target space geometry. The interplay between these two-dimensional gauge theories and the quantum geometry on the target space offers a far-reaching connection between two-dimensional gauge theories and their dualities on the one hand and Gromov–Witten theory and mirror symmetry on the other hand [3,4].

The aim of this work is to systematically study the structure and the underlying geometry of a certain class of correlators of both Abelian and non-Abelian two-dimensional $\mathcal{N} = (2, 2)$ supersymmetric gauged linear sigma models, which depend on the Fayet–Iliopoulos parameters and the theta angles of the gauge theory. The recent work [5] by Closset, Cremonesi and Park furnishes an important ingredient in our approach, as it offers techniques to exactly compute these correlators of gauged linear sigma models — generalizing the methods of Morrison and Plesser [2] to higher point correlators and to non-Abelian gauged linear sigma models.[1] Their approach is based upon modern localization techniques of supersymmetric gauge theories on curved spaces with a non-trivial (off-shell) supergravity background [7,8], such that the quantum–exact correlators localize on a sum of non-trivial topological vortex sectors. As the performed localization calculation in the specified supergravity background directly relates to similar computations by Hori and Vafa in the context of A-twisted gauged linear sigma models on the symplectic side of mirror symmetry [4], these correlators contain information about the quantum Kähler moduli space and the Gromov–Witten theory of the target space.

In this note — starting from the residue integral of the localized gauge theory correlators provided in ref. [5] — we derive universal and non-trivial relations among the set of all gauge theory correlators, which are directly and easily obtained from the spectrum of the gauge theory. Giving a Hilbert space interpretation for the correlators, we map the obtained universal relations to differential operators that annihilate the ground state of the gauge theory. Realizing the gauge theory correlators as certain quadratic pairings of the Givental I-function — as argued by Ueda and Yoshida [9] and proven for a particular class of target space geometries and conjectured more generally in ref. [10] — we argue that the obtained set of differential operators generates the GKZ system of differential equations governing the quantum cohomology of the target space.[2] As a consequence, the obtained differential operators are in agreement with differential equations for the quantum periods of the A-twisted gauged linear sigma models studied in ref. [4].[3]

For the important class of two-dimensional $\mathcal{N} = (2, 2)$ supersymmetric gauged linear sigma models with a non-anomalous axial $U(1)_R$ R-symmetry, the gauge theories are known to flow in the IR to non-trivial families of two-dimensional $\mathcal{N} = (2, 2)$ superconformal field theories [1], where the Fayet–Iliopoulos parameters and the theta angles furnish the algebraic coordinates

[1] See also ref. [6] for correlators of two-dimensional $\mathcal{N} = (2, 2)$ gauged linear sigma models.

[2] For a connection between the Givental I-function of the target space and the vortex partition function of two-dimensional $\mathcal{N} = (2, 2)$ quiver gauged linear sigma models see also ref. [11].

[3] For particular examples of gauge theories, Closset et al. similarly deduce differential equations from the localized gauge theory correlators, which in our approach arise from universal correlator relations.