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Invariant operators, orthogonal bases and correlators in general tensor models

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Abstract

We study invariant operators in general tensor models. We show that representation theory provides an efficient framework to count and classify invariants in tensor models of (gauge) symmetry $G_d = U(N_1) \otimes \cdots \otimes U(N_d)$. As a continuation and completion of our earlier work, we present two natural ways of counting invariants, one for arbitrary G_d and another valid for large rank of G_d. We construct bases of invariant operators based on the counting, and compute correlators of their elements. The basis associated with finite rank of G_d diagonalizes the two-point function of the free theory. It is analogous to the restricted Schur basis used in matrix models. We show that the constructions get almost identical as we swap the Littlewood–Richardson numbers in multi-matrix models with Kronecker coefficients in general tensor models. We explore the parallelism between matrix model and tensor model in depth from the perspective of representation theory and comment on several ideas for future investigation.

1. Introduction

Tensor models, whose elementary building block consists of tensorial objects, provide a natural generalization of matrix models. In theoretical physics, there are various motivations that

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make the tensor model an interesting system to study. In one corner, the motivation comes from a scheme for studying quantum entanglement. From the quantum mechanical point of view, d-rank tensor models are associated with the multi-linear symmetry group $G_d(\mathbf{N}) = U(N_1) \otimes U(N_2) \otimes \cdots \otimes U(N_d)$ acting on a tensor product Hilbert space $\mathcal{H} = \mathcal{H}_{N_1} \otimes \cdots \otimes \mathcal{H}_{N_d}$. We know that the Hilbert space of a composed physical system is the tensor product of its constituents, and quantum correlation among them is an essential aspect of entanglement in quantum mechanics [1]. So tensor models naturally describe composite systems. Moreover, gauge invariant operators built out of tensors separate the entangled and untangled states of $\mathcal{H}$, so they can be viewed as a probe of quantum entanglement measurements [2].

In another corner, tensor models provide a suitable scheme for studying quantum gravity. Inspired by the success of matrix models in describing two-dimensional quantum gravity [3], tensor model was proposed as a framework for describing higher-dimensional random geometry [4–6]. Colored tensor models [7,8] and the development of their $1/N$-expansion [9–11] have triggered an upsurge of the subject and a fast growth in recent years. The introduction of color has served to overcome several difficulties that the earlier tensor models had in describing quantum gravity at dimensions greater than two. More recently, the colored tensor model have been found in direct connection with the AdS_2/CFT_1 holography, as an alternative formulation of the Sachdev–Ye–Kitaev (SYK) model [12–21] in which the necessity of quenched disorder is dispensed while exhibiting the same large-N behavior [22], see also [23]. Tensor models were also studied in the non-perturbative definition of non-abelian tensor fields [24], where interesting connections with matrix factorizations and dynamical Yang–Baxter maps were found.

The simplest yet nontrivial tensor model is the matrix model, which has been studied extensively in the context of AdS/CFT correspondence. In the matrix model, the use of orthogonal bases for two-point functions (first for the BPS-sector [25] and then for general bosonic sectors [26–32] and for including gauge field [33] or fermions [34]) (see also [35])) was extremely useful for computations in $\mathcal{N} = 4$ super Yang–Mills theory within the so-called non-planar regime, which involves heavy operators dual to excited D-branes and macroscopic solitonic objects in the string theory side [36–40].

In all these situations, the colored tensor model is considered as a n-dimensional quantum field theory (which, as originally envisioned, may eventually describe spacetime and matter in $D \geq n$ dimensions), where the fundamental degrees of freedom are tensor fields transforming as a suitable (not necessarily irreducible) representation under an internal symmetry G_d. While there are issues of the tensor model pertinent to the quantum field theory such as renormalizability [41], there are also issues associated with the internal symmetry G_d that need to be understood first. These issues are largely related to the representation theory, so we will for simplicity take the colored tensor model to be zero-dimensional. The aim of this paper is to undertake detailed study of this zero-dimensional tensor model, expanding our earlier work [42].

This paper is meant to be a comprehensive revision and completion of our earlier work [42]. Thus, there is a significant overlap with the first paper. Nevertheless, the present work ties up all the loose ends of the former by adding new proofs (like eq. (3.19) which shows the match between the finite and the large N operator counting, or the orthogonality of the proposed operator basis in all the labels by direct computation of the correlators in eq. (5.15)), further examples and clarifications. Section 6 is also new.

The paper is organized as follows. We first recapitulate aspects of basic representation theory relevant for analysis in later sections. We then count physical observables, viz. invariants of tensor fields, in section 3, following the steps of [43] and [44]. Kronecker coefficients appear naturally in the counting. We show that representation theory actually provides two natural ways

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